E-loss in cold nuclear matter (a jet quenching lab.)

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Thanks to Alberto for the introduction!

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Two basic related questions

1) How is jet structure modified by a medium

1b) test of LPM effect in QCD

1c) Deeper understanding of soft radiation DL regime ?

2) What can be learnt about the medium from modification

2b) momentum transfer distributions form medium to jet

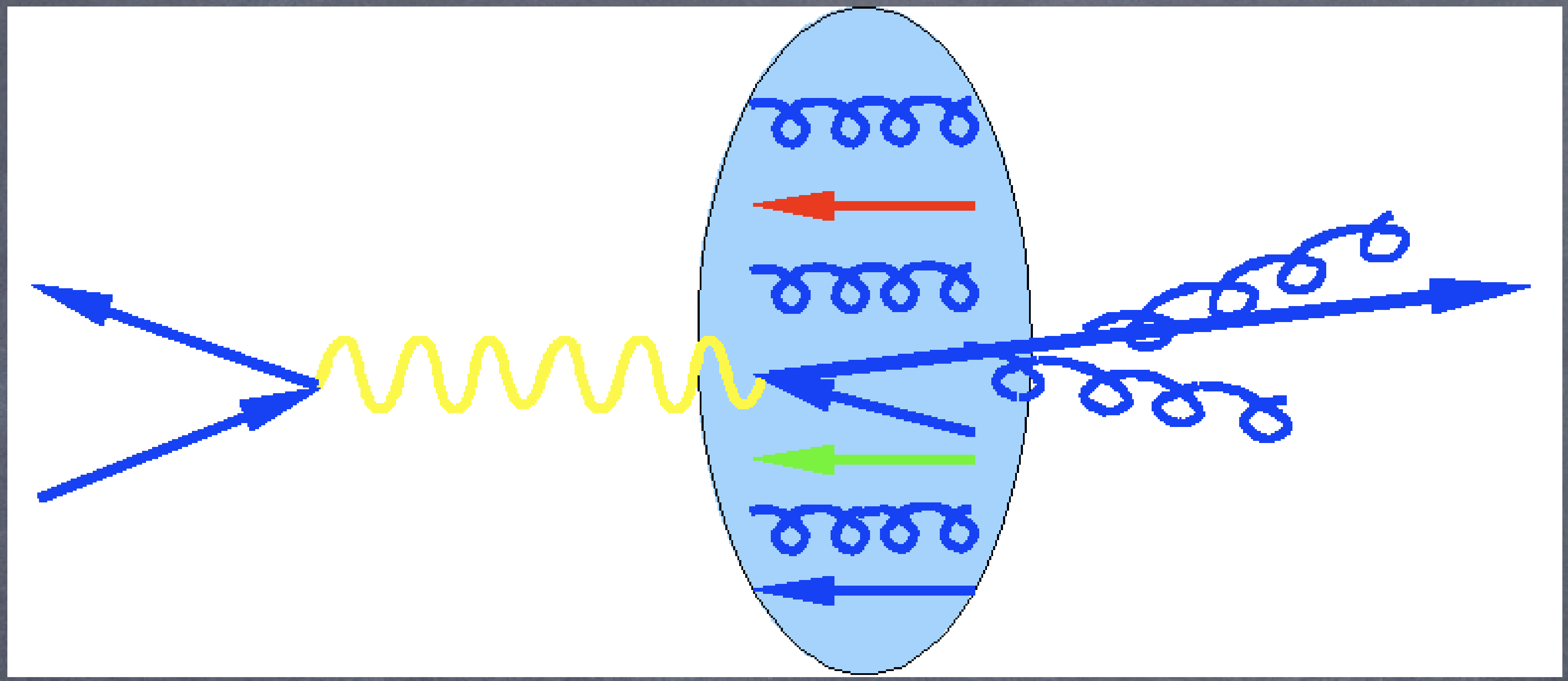
2c) can we use moments $\hat{q}$, $\hat{e}$,$\hat{e}_2$ etc.

2d) constrains on known bulk structure.

2e) To saturate or not to saturate

What is the experiment

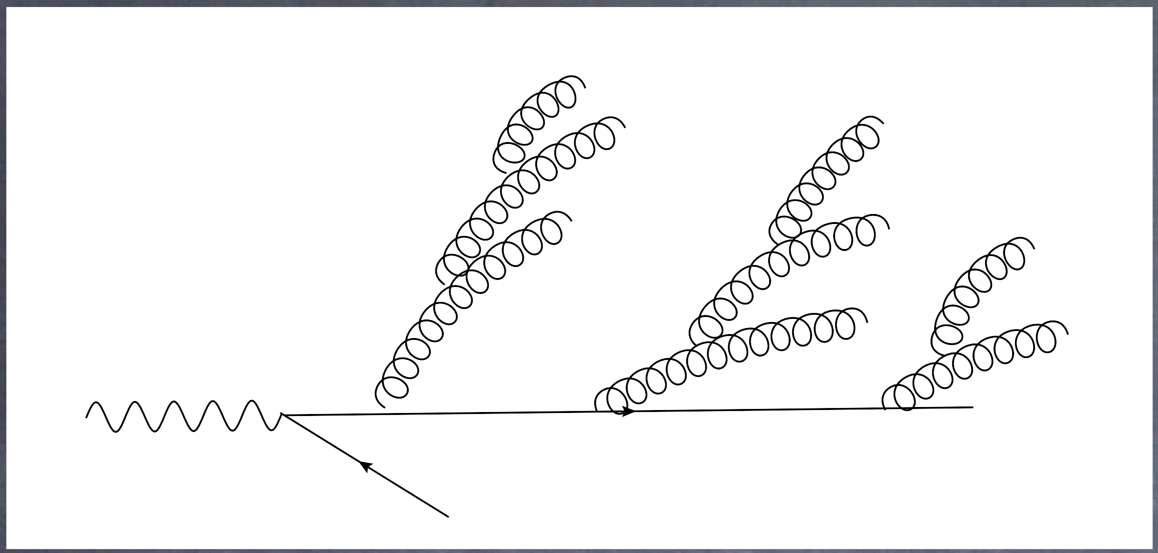
DIS with a hard jet in the final state

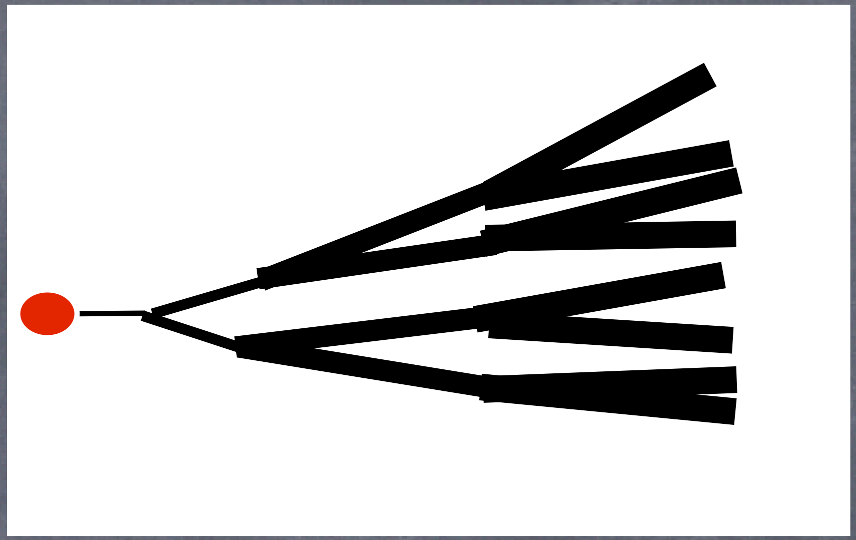


Note: the produced jet (not the photon) is the probe

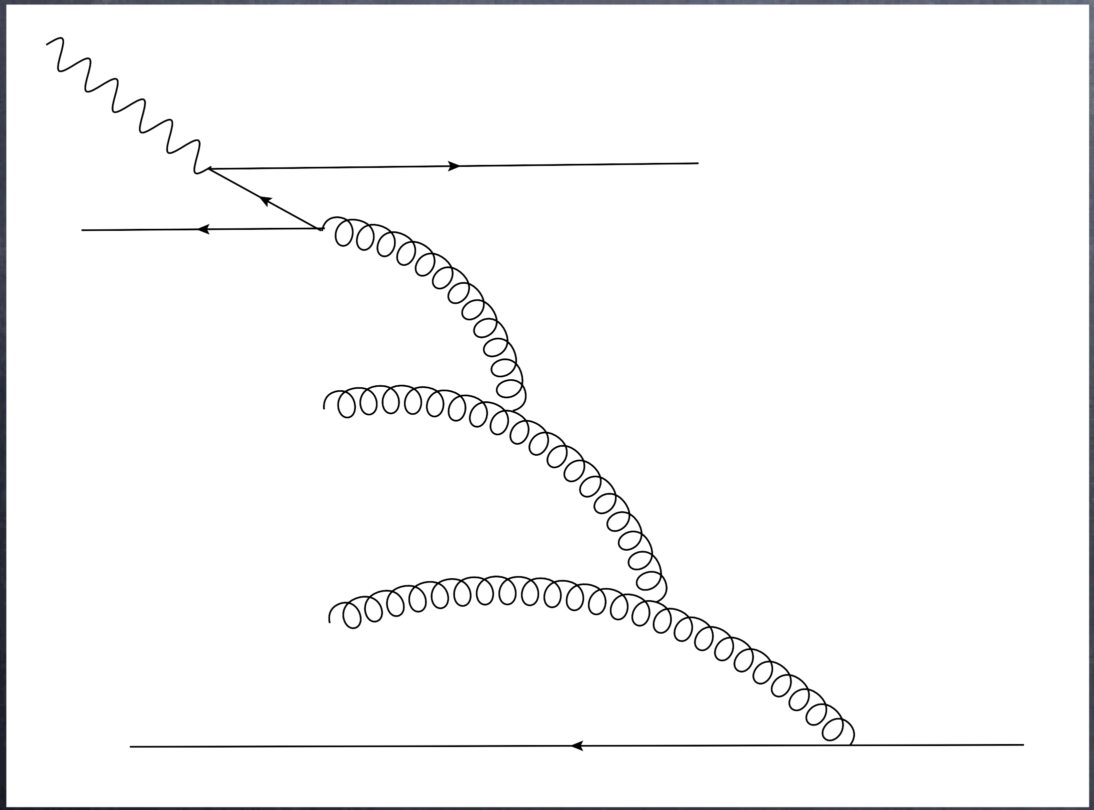
In order to study this partonically:
the jet scale (virtuality) has to be hard on entry and exit

why? perturbative control only at large Q

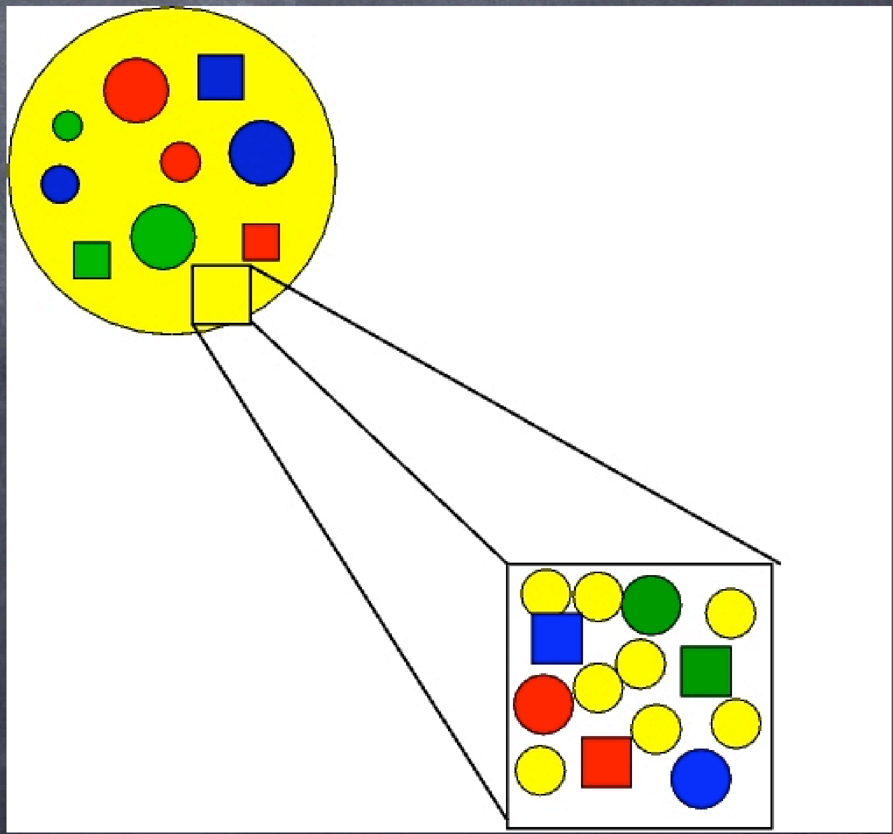




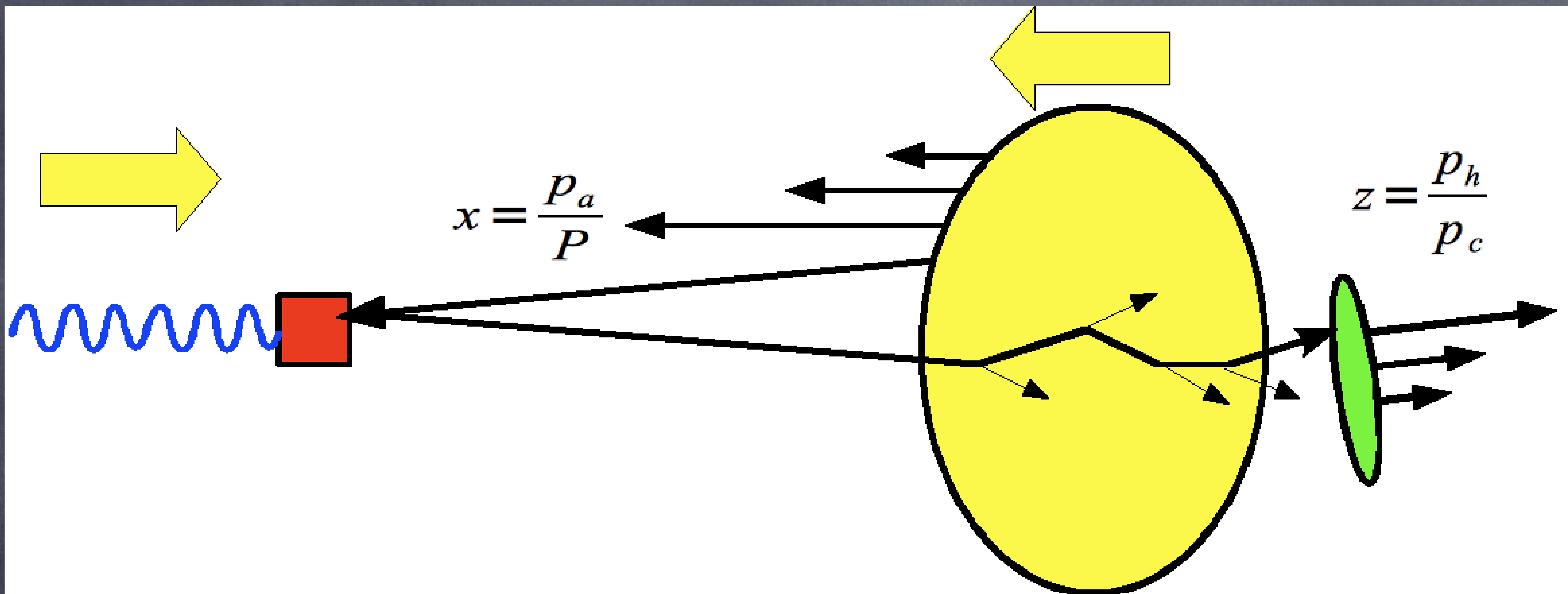
In the absolute Breit frame
This leads to a scale dependent resolution



always scatter off the partonic substructure



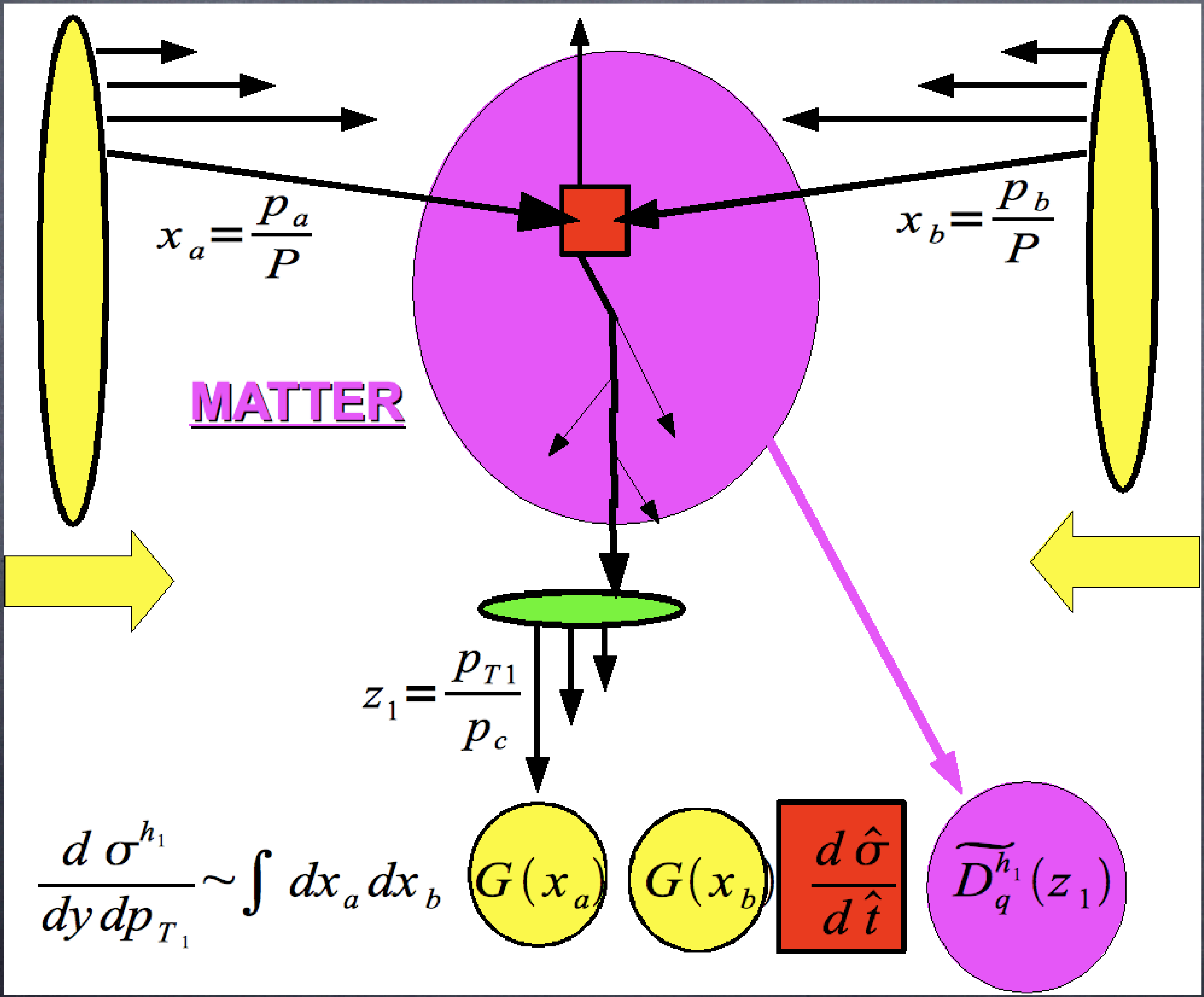
Large scale allows for a factorized approach



$$d\sigma^{h_1} \sim \int dx \; G(x) \; d\hat{\sigma}(x, q, Q^2) \; \widetilde{D_q^{h_1}}(z_1)$$

$$\tilde{D} = D + \text{pQCD corrections} * F(x)$$

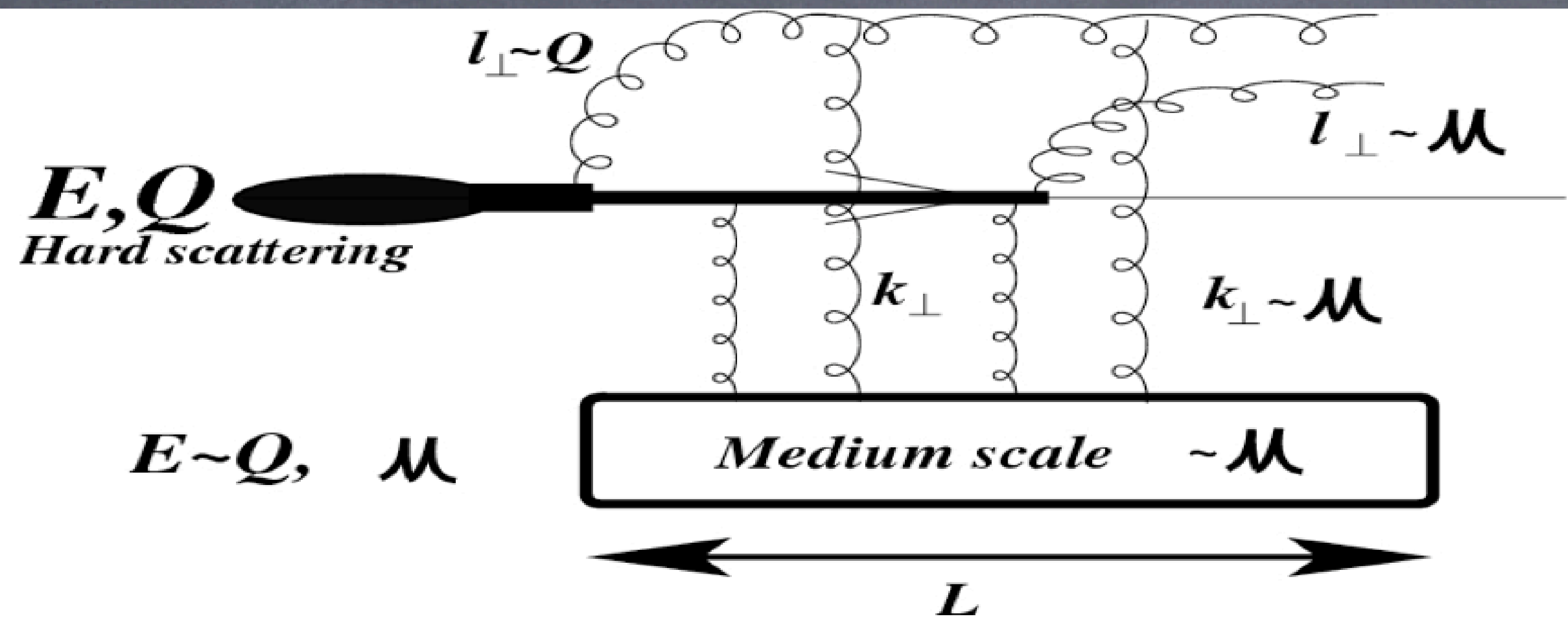
Given factorization and universality, Can connect e^+e^-, DIS, pp and HI collisions



Possibility of setting up a rigorous theory

at some large Q, compare directly with experiment

no fudge!



Jet forward energy: $E, q^- \sim Q \gg m_J \gg \Lambda_{QCD}$ ***mass of proton,***

Virtuality of photon: $Q \gg l_\perp \leqslant m_J$ ***Virtuality of jet,***

Radiated gluon momentum: $\left[\dfrac{l_\perp^2}{2q^- y}, yq^-, l_\perp \right]$

Soft medium gluons $\lambda_{QCD} \ll k_\perp \ll l_\perp$ ***However!*** $A^{\frac{1}{3}} k_\perp \leqslant l_\perp$

$L \sim A^{\frac{1}{3}}$ ***A, atomic number of the nucleus,***

Notation

$$p^{+} = (p^{0} + p^{3})/\sqrt{2}$$

$$q^{-} = (q^{0} - q^{3})/\sqrt{2}$$

$$p \cdot q = p^{+} q^{-} + p^{-} q^{+} - p_{\perp} \cdot q_{\perp}$$

Everybody in nucleus direction has large p^+

Everybody in photon direction has large q^-

$$x_B = \frac{Q^2}{2\, p^{+} q^{-}} \equiv \frac{Q^2}{2\, M \nu}$$

In Breit frame *In target frame*

$$z = \frac{p^{-}_{h,\,Breit}}{q^{-}} \equiv \frac{p_{h,\,target}}{\nu}$$

Can we set up a well defined effective theory?

Need a small parameter, not g ~ 2, not α_s ~ 0.3

Take a cue from Soft Collinear Effective theory
the ratio of virtuality to jet energy (m_J and E)
call this λ !

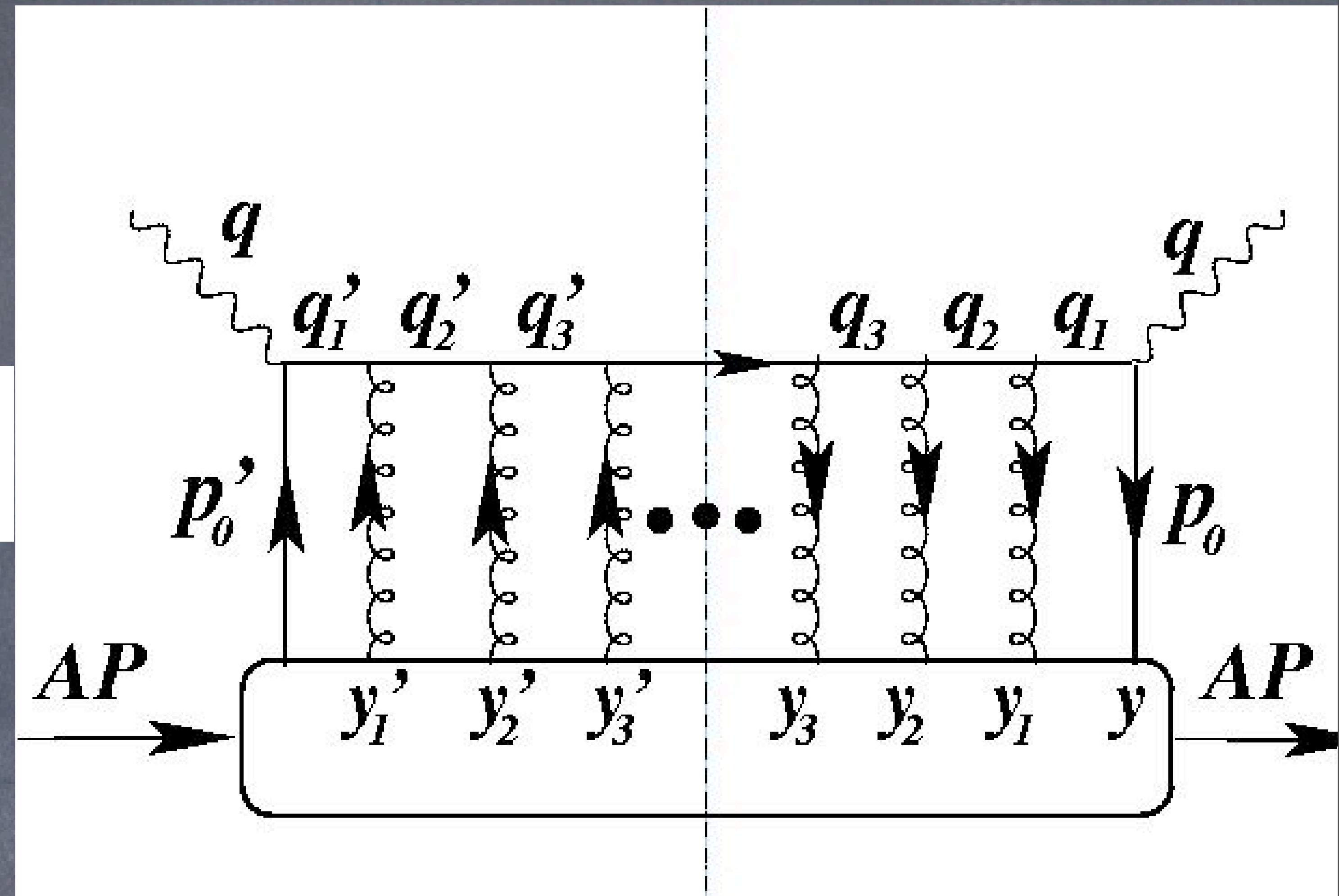
Pretty much everything is controlled by jet virtuality

Organize the whole calculation in λ

How is a single hard parton modified

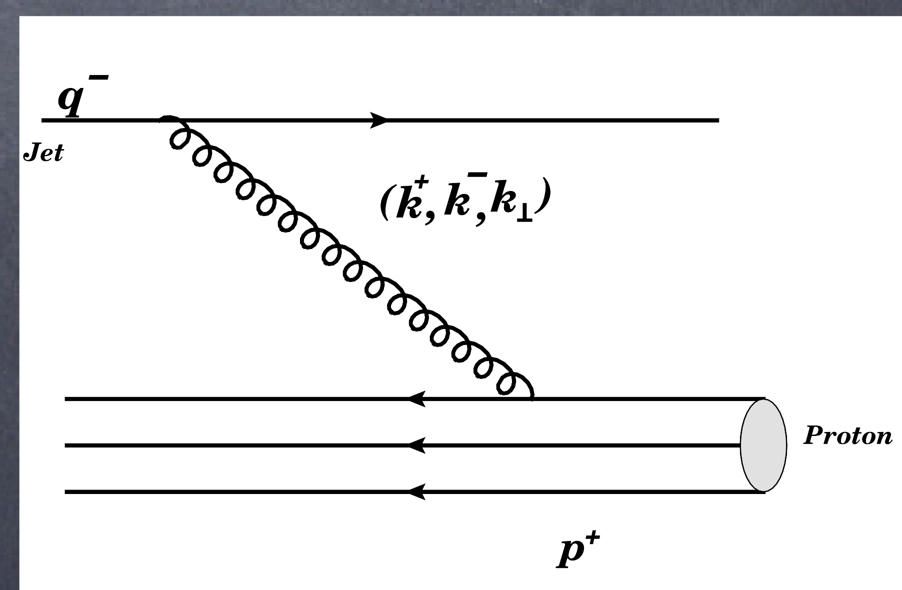
$$Photon\ has,\quad q \equiv \left[\frac{-Q^2}{2q^-}, q^-, 0, 0\right]$$

$$quark\ has,\quad p_0 = \left[x_B P^+, 0,0,0\right],\ x_B = \frac{Q^2}{2p^+ q^-}$$



Struck quark has,

energy ~ Q and
virtuality ~ λQ



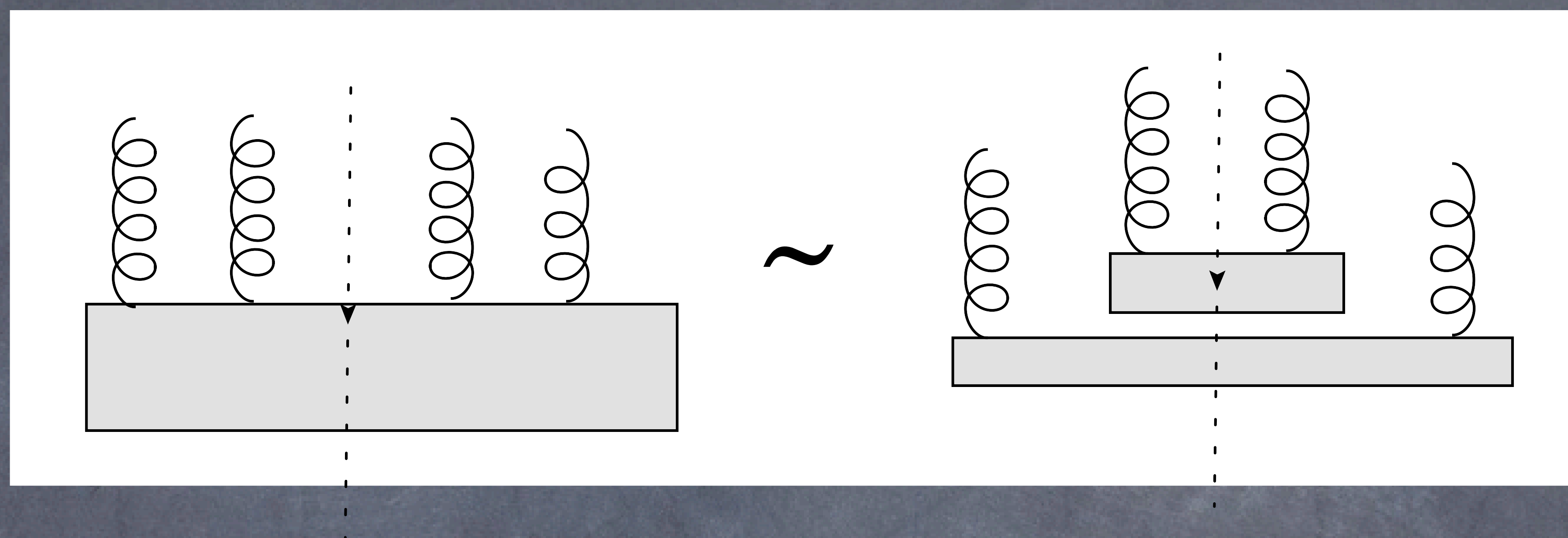
hence, gluons have

$$k_\perp \sim \lambda Q, \qquad k^+ \sim \lambda^2 Q$$

could also have $k^- \sim \lambda Q$

Calculate in negative light-cone gauge $A^- = 0$

Take the extreme limit of a nucleus, A -> ∞ and nucleons are very small compared to nucleus



All four gluons from one nucleon: prop. to L

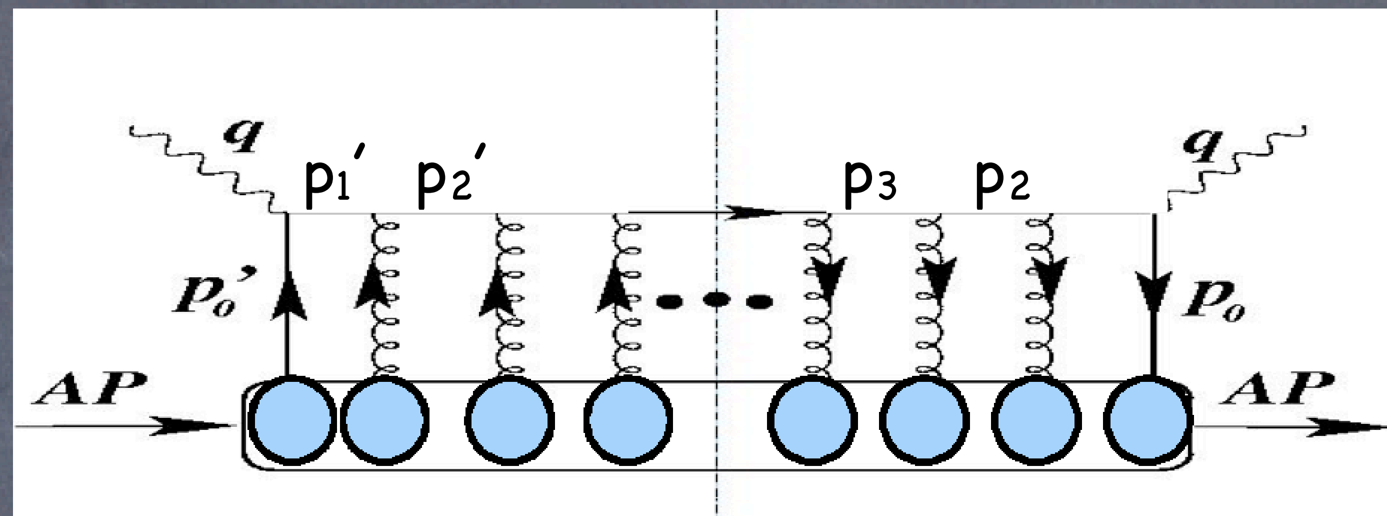
Two in one nucleon, two in another: prop. to L^2

2 n gluon expection ---> n 2 gluon expectation

So what do we get from resumming ?

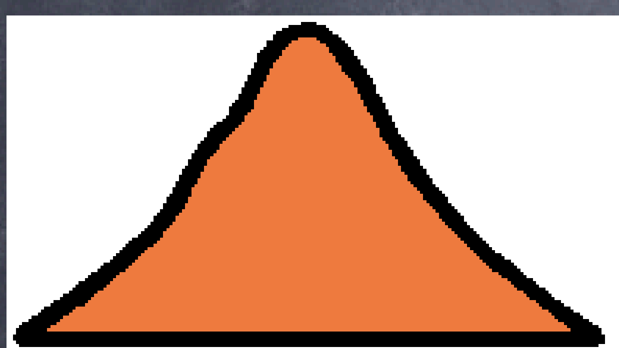
a) transverse broadening

$$p^+ = \frac{p^0 + p_z}{\sqrt{2}}$$

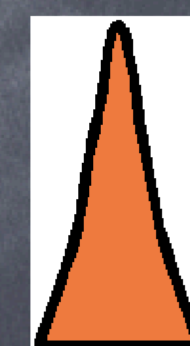


$$p^- = \frac{p^0 - p_z}{\sqrt{2}}$$

Assuming independent scattering off nucleons gives a diff. equation



$$\frac{\partial f(p_\perp, t)}{\partial t} = \nabla_{p_\perp} \cdot D \cdot \nabla_{p_\perp} f(p_\perp, t)$$



$$\langle p_\perp^2 \rangle = 4Dt$$

$$\hat{q} = \frac{p_\perp^2}{L^-} = \frac{4\pi^2 \alpha_S C_R}{N_c^2 - 1} \int \frac{dy^-}{2\pi} e^{-i\left(\frac{k_\perp^2}{2q^-} y^- - k_\perp \cdot y_\perp\right)} \langle F^{\mu\alpha} v_\alpha(y^-, y_\perp) F_\mu^\beta(0) v_\beta \rangle$$

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\hat{q} = \frac{p_\perp^2}{L^-}
= \frac{4 \pi^2 \alpha_S C_R }{ N_c^2 - 1 } \int \frac{d y^-}{2 \pi} e^{ -i
\left( \frac{k_\perp^2}{2 q^-} y^- - k_\perp \cdot y_\perp \right) } \langle
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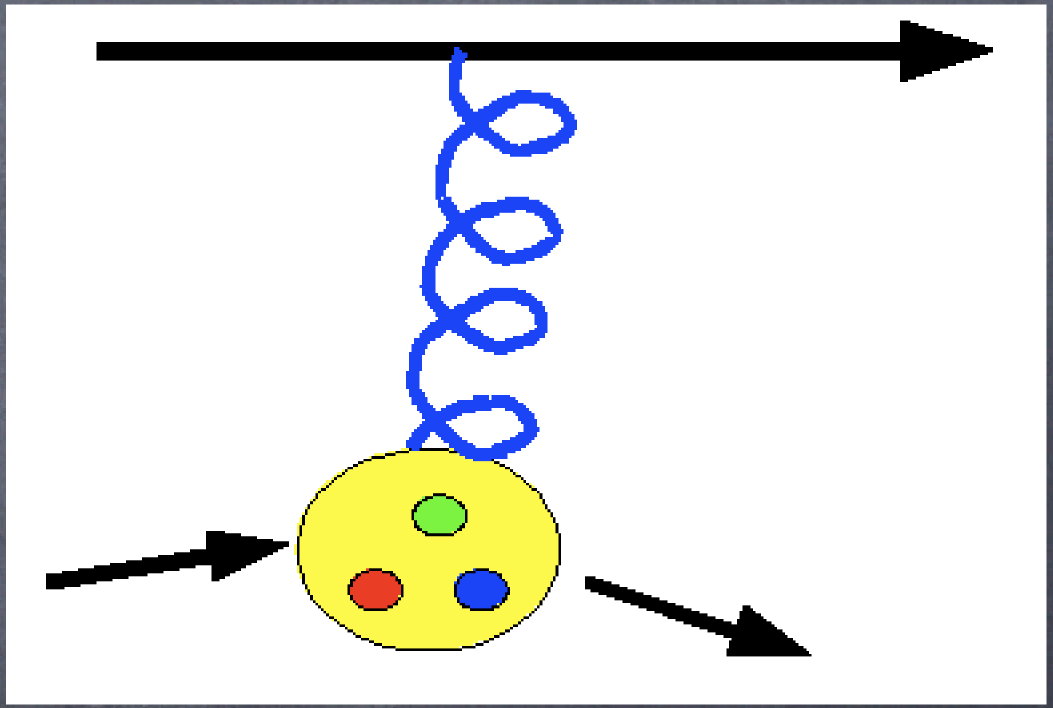
b) Longitudinal drag and diffusion

A close to on shell parton has a 3-D distribution

$$p^+ = \frac{p_\perp^2}{2p^-}$$

$$f(\vec{p}) \equiv \delta^2(p_\perp^2)\delta(p^- - q^- + k^-)$$



Using the same analysis, we get a drag. and diff. term

$$\frac{\partial f(p^-, L^-)}{\partial L^-} = c_1 \frac{\partial f}{\partial p^-} + c_2 \frac{\partial^2 f}{\partial^2 l^-}$$

c_1 is dE/dL, calculate in a deconfined quasi-particle medium.

What have we done by reducing to transport coefficients ?

There is some distribution of 4-momentum transfer

replace with width (variance)

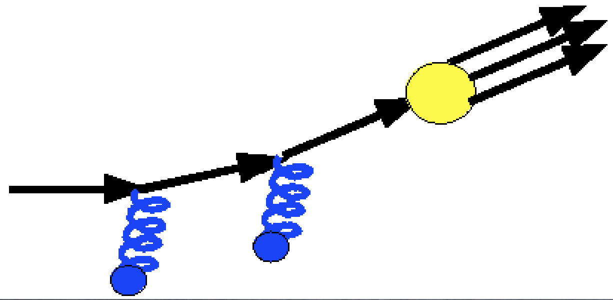
there is also a k^- and a k^+ distribution,

k^- gives elastic loss, with drag and diffusion, replaced with mean and variance

k^+ gives virtuality generation, integrated over, peak at $$k^+ = \frac{k_\perp^2}{2q^-}$$

There are a bunch of medium properties which modify the parton and frag. func.

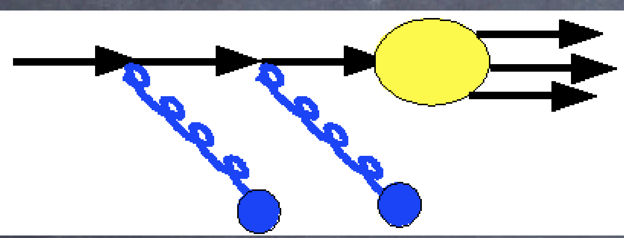
$\hat{q}$, $\hat{e}$ = dE/dL and $\hat{f}$ = dN/dL



$$D\left(\frac{\vec{p}_h}{|\vec{p}+\vec{k}_\perp|}, m_J^2\right)$$

$$\hat{q} = \frac{\langle p_T^2 \rangle_L}{L}$$

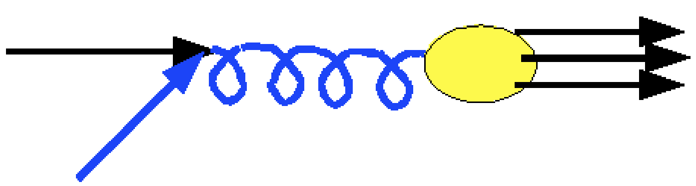
Transverse momemtum diffusion rate



$$D\left(\frac{p_h}{p-k}, m_J^2\right)$$

$$\hat{e} = \frac{\langle \Delta E \rangle_L}{L}$$

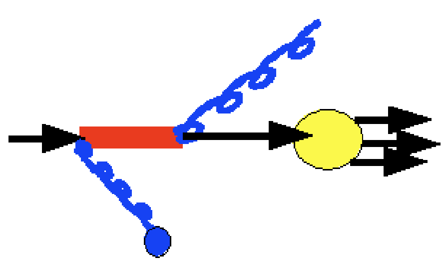
Elastic energy loss rate also diffusion rate e_2

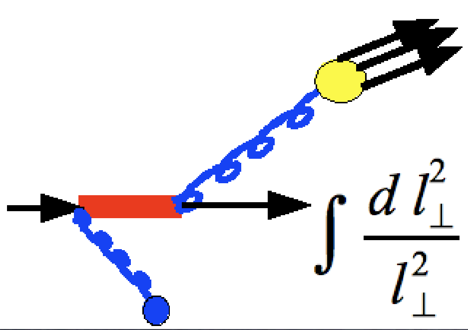


$$D_g\left(\frac{p_h}{p+k}, m_J^2\right)$$

$$\hat{f} = \frac{\langle \Delta N \rangle_L}{L}$$

Flavor (q <-> g) diffusion rate





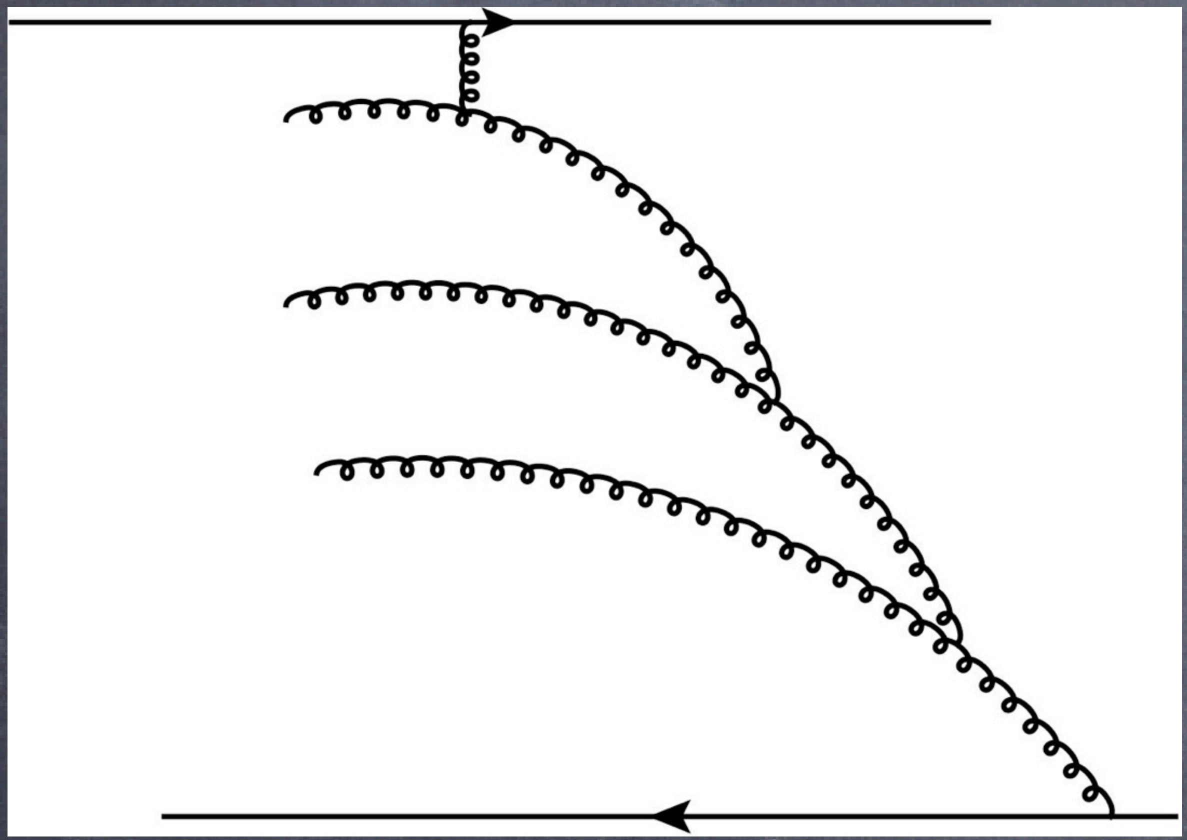
$$\int \frac{d\,l_\perp^2}{l_\perp^2} \int_{\frac{p_h}{p}}^{1} \frac{d\,y}{y} P(y)\, M(\vec{r}, y, l_\perp)\, D\left(\frac{p_h}{p\,y}\right)$$

Gluon radiation is sensitive to all these transport coefficients

And a bunch of off diagonal and higher order transport coefficients

An aside on the calculation of transport coeff.

Since the jet is at a high virtuality
The q or e will have to be evolved
up in Q^2 and down in x



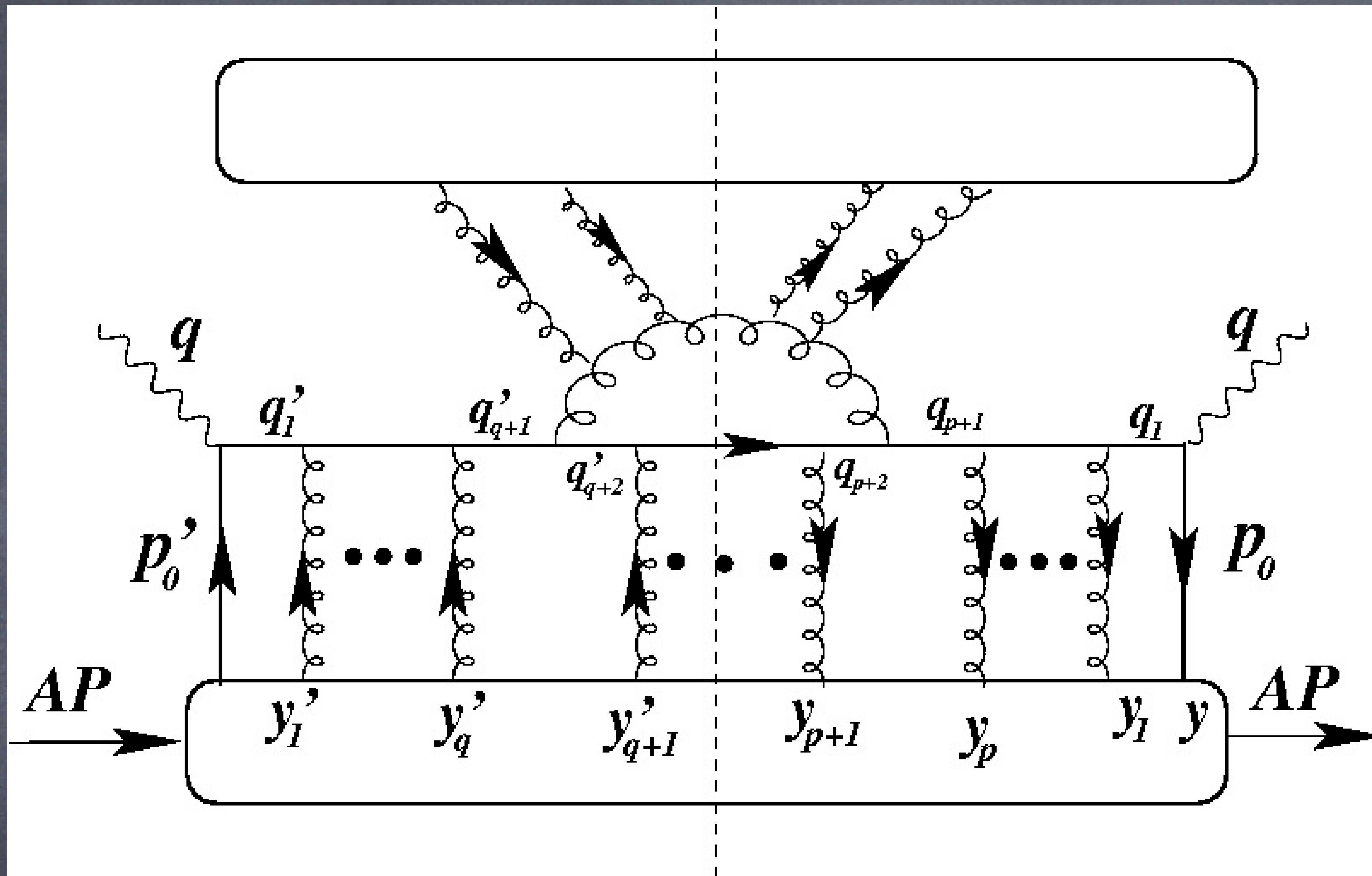
$$x = \frac{k_\perp^2}{2p^+q^-} = \lambda^2$$

x = \frac{k_\perp^2}{2 p^+ q^-} = \lambda^2

$$Q^2 \sim k_\perp^2 \sim l_\perp^2 \sim m_J^2$$

Can this be calculated in the CGC picture
does the small x distribution spill over into the next nucleon ?
Yes! cannot consider nucleons separately,
No, then separate nucleons o.k.
but do you need non-pert. input to confine or does CGC confine by itself

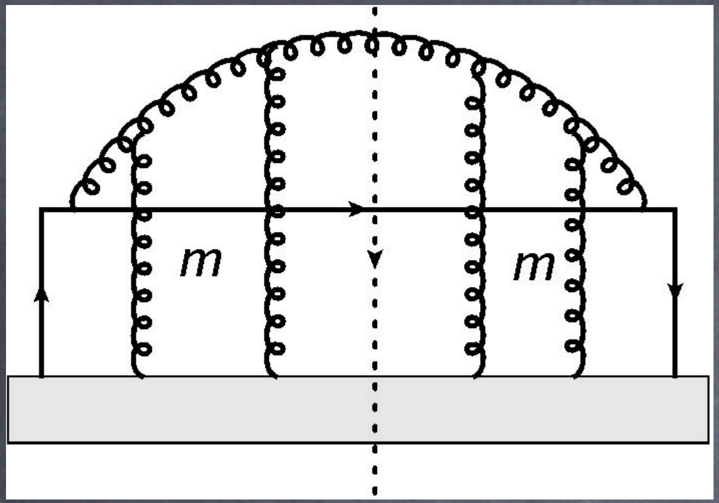
The single gluon emission kernel



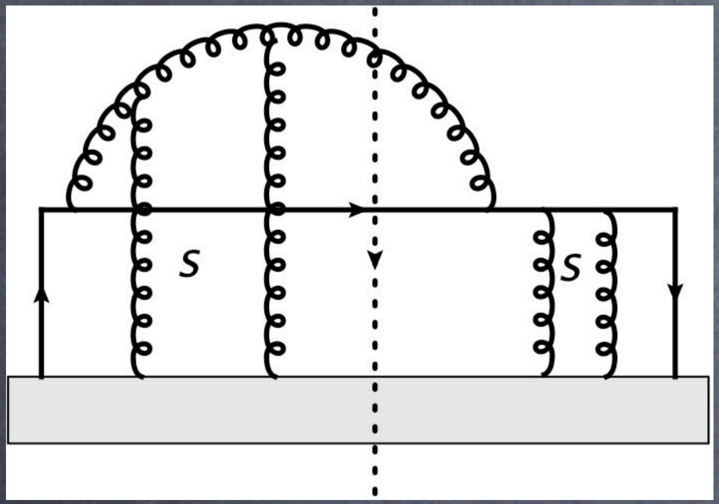
Calculate 1 gluon emission with quark & gluon N-scattering with transverse broadening and elastic loss built in

Finally solved analytically, in large Q^2 limit.

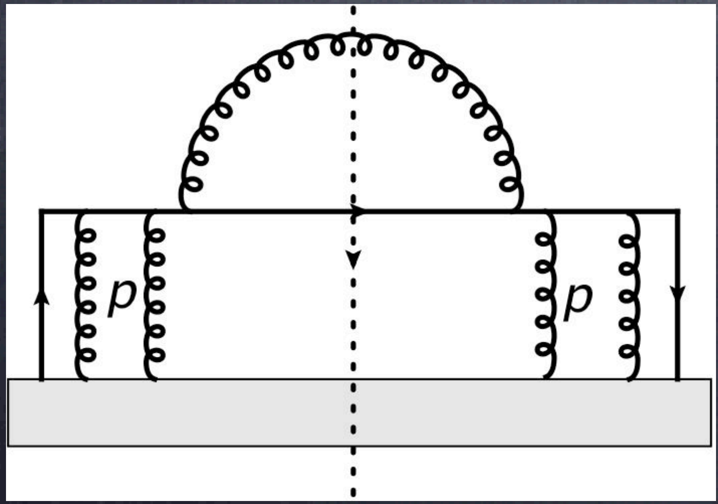
The different non-zero contributions



$$\sim C_A^m \int dy \frac{dl_\perp^2}{l_\perp^2} P(y) \int d\zeta^- \frac{2\hat{q}(\zeta^-)}{l_\perp^2} \left[2 - 2\cos\left(\frac{l_\perp^2}{2q^-} \zeta^- \right) \right]$$

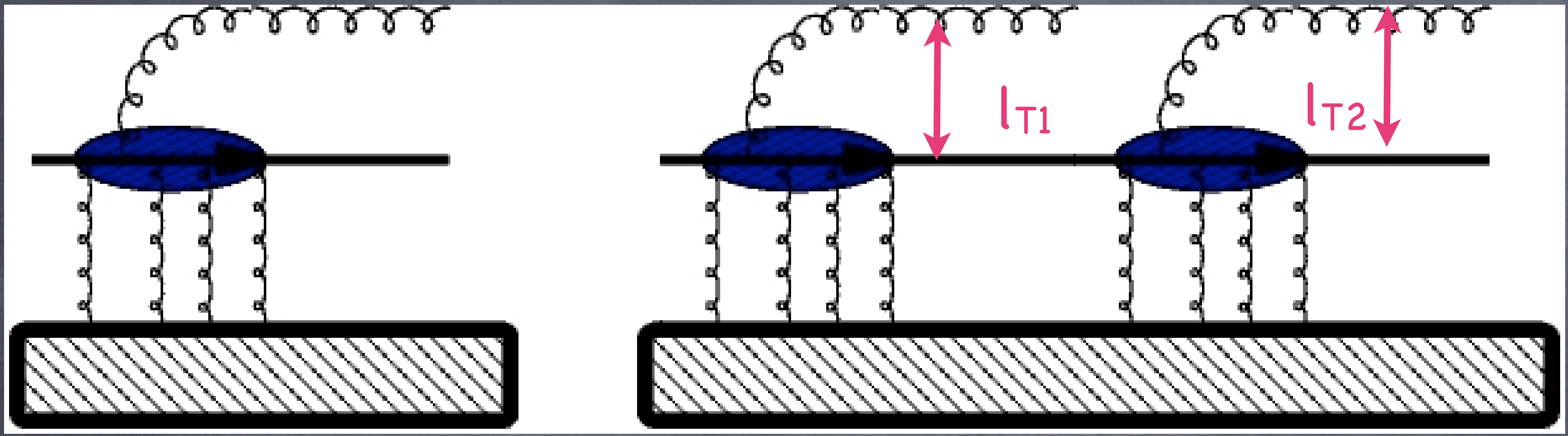


$$\sim -\left(\frac{C_A}{2} \right)^s \int dy \frac{dl_\perp^2}{l_\perp^2} y P(y) \int d\zeta^- \frac{\hat{q}(\zeta^-)}{2 l_\perp^2} \left[2 - 2\cos\left(\frac{l_\perp^2}{2q^-} \zeta^- \right) \right]$$



$$\sim -(C_F)^p \int dy \frac{dl_\perp^2}{l_\perp^2} y^2 P(y) \int d\zeta^- \frac{\hat{q}_Q(\zeta^-)}{l_\perp^2} \left[2 - 2\cos\left(\frac{l_\perp^2}{2q^-} \zeta^- \right) \right]$$

Need to repeat the kernel



What is the relation between subsequent radiations ?

In the large Q^2 we can argue that there should be ordering of l_T.

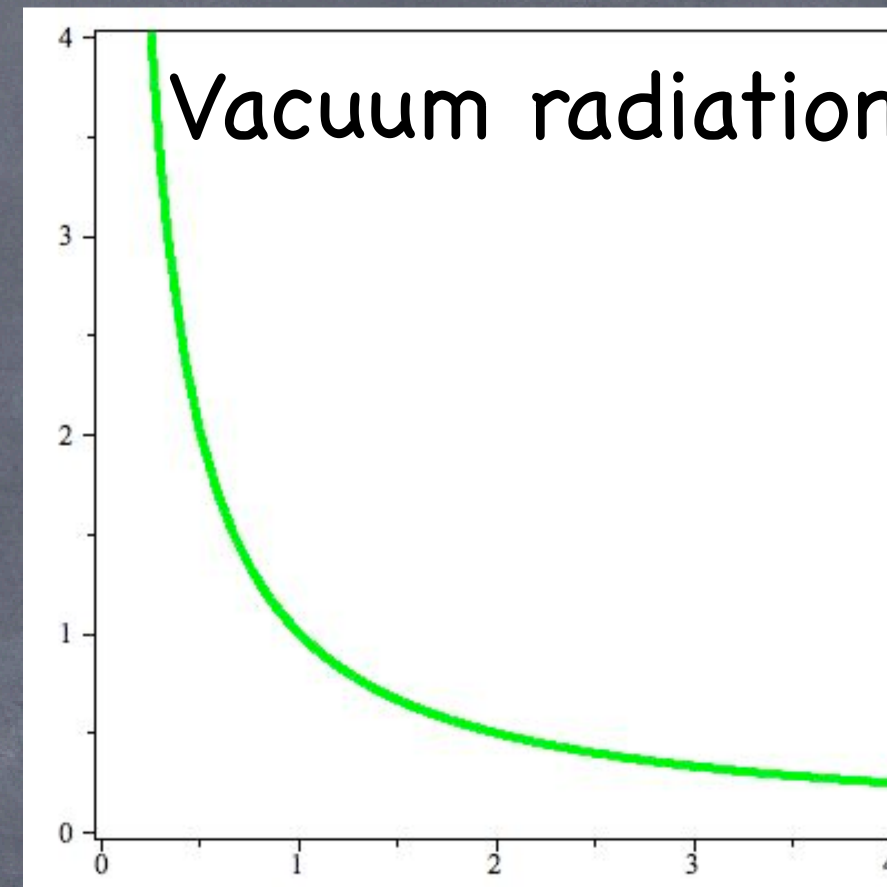
The same statement in a plot

Virtuality is like $l_\perp^2$, At leading log, CS goes as $dl_\perp^2/l_\perp^2$

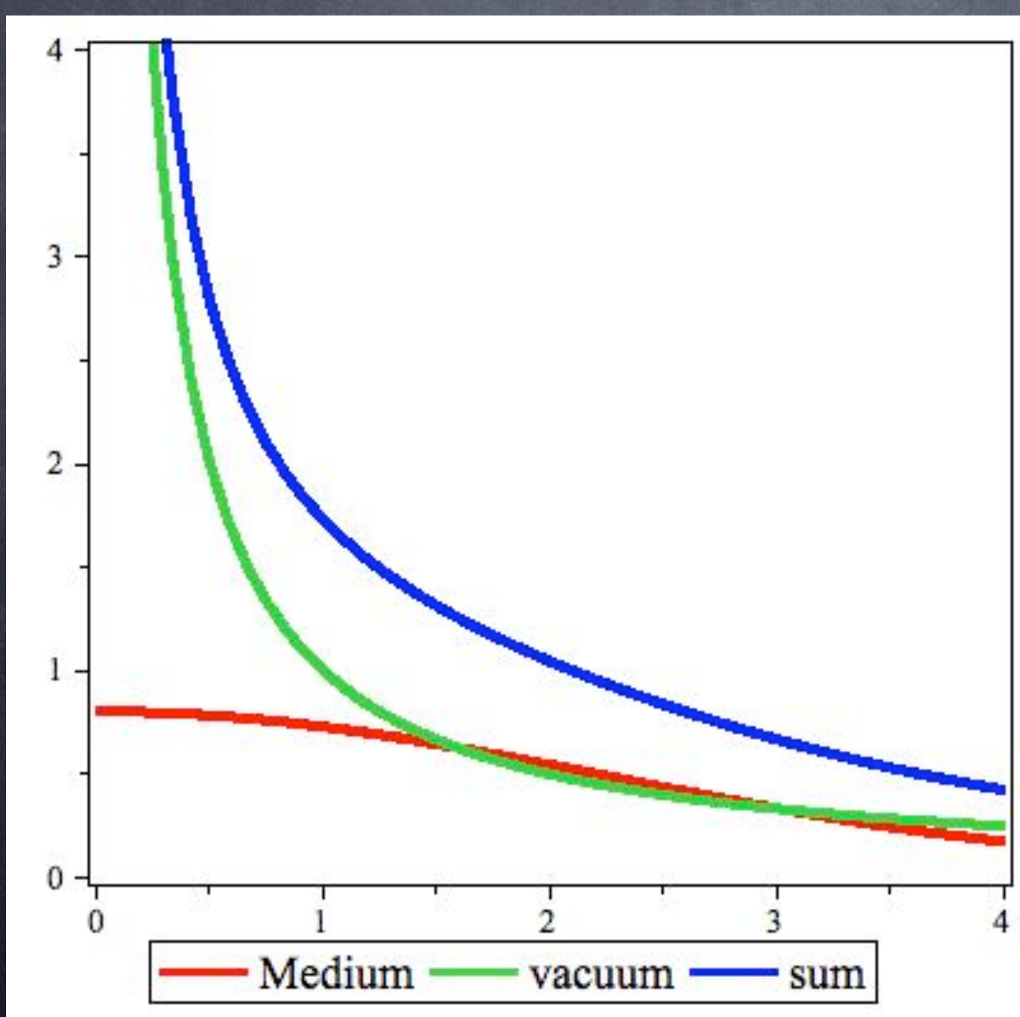
Integrating over this yields a $\log(\mu_1^2/\mu_2^2)$

Multiple emissions will yield large logs if strongly ordered

$\frac{1}{l_\perp^2}$



$l_\perp^2$



This CS is slightly modified in the medium

Include the largest correction from the medium

dσ = Log + # L,

If form is not too different, then sum with DGLAP

Testing this ordering of radiation

Assuming nuclear p. d. f. = A X nucleon p. d. f.

we can construct the ratio of the frag. funcs.

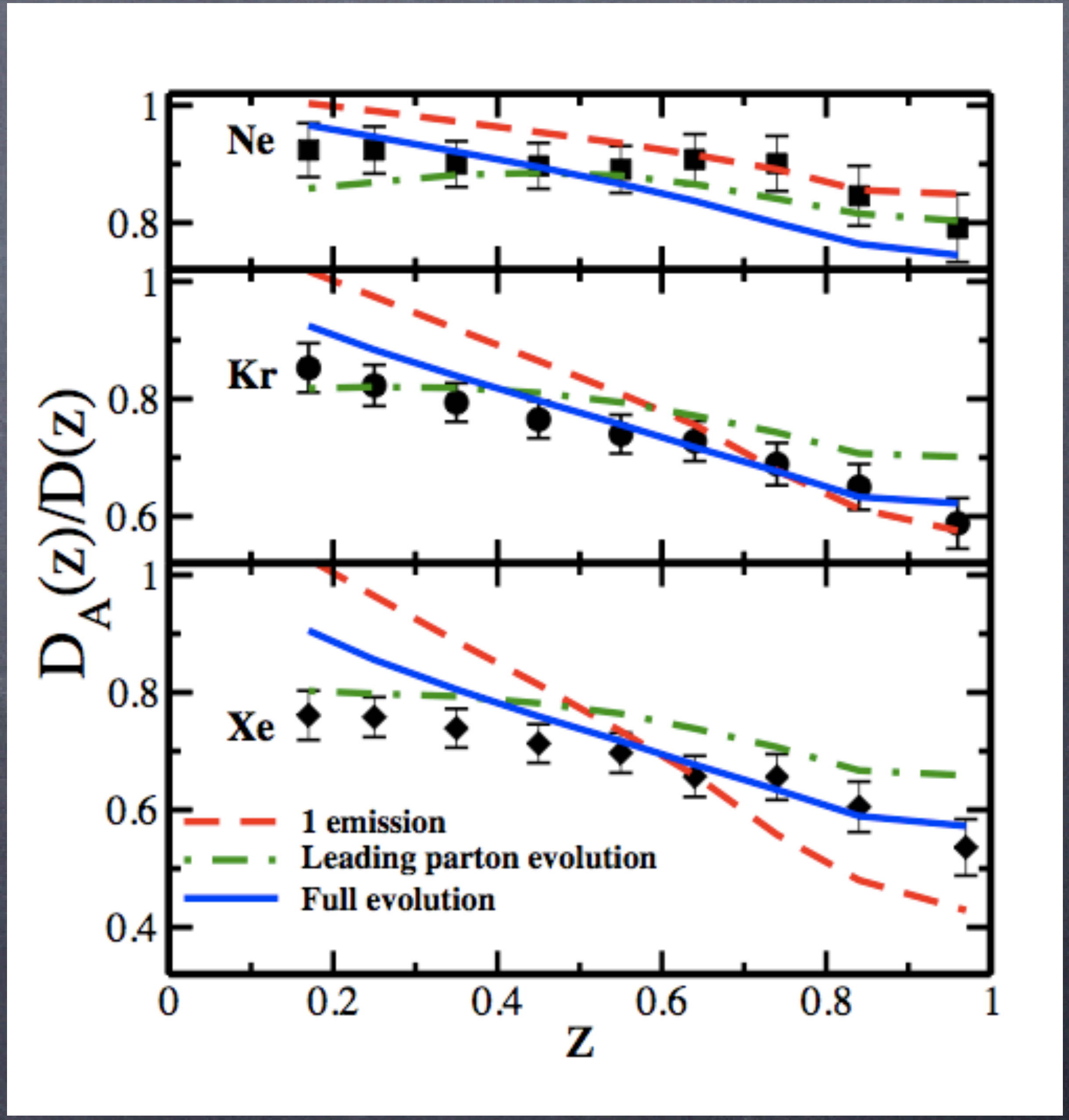
Data from HERMES at DESY

Three different nuclei

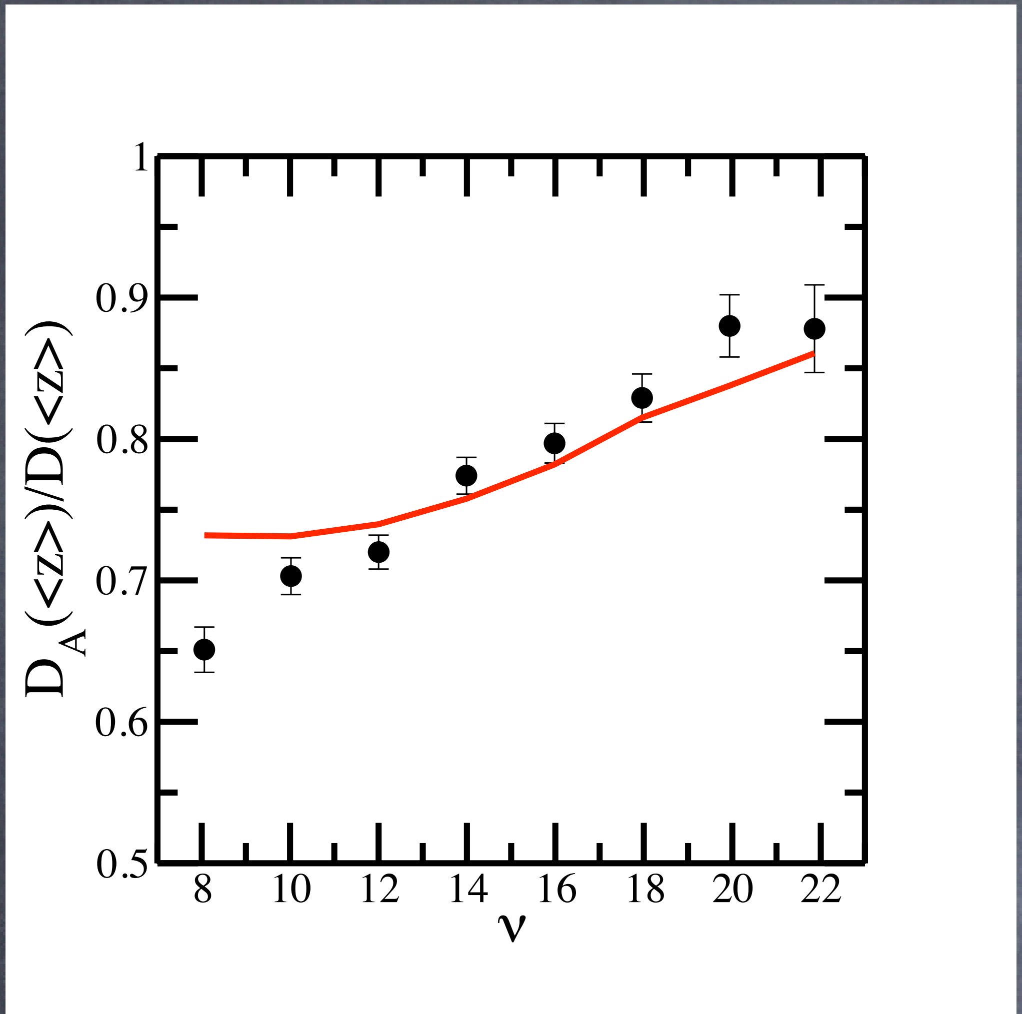
one $\hat{q}$ = 0.08GeV^2/fm

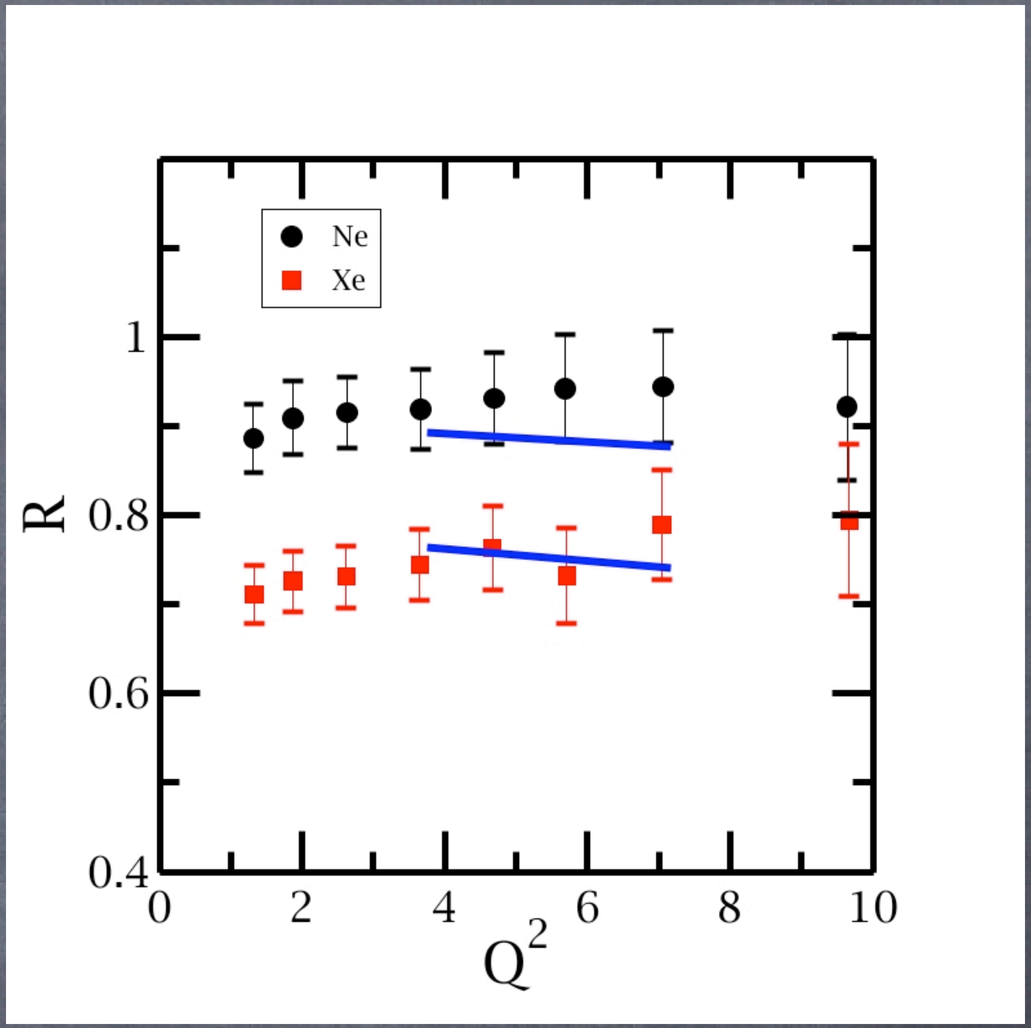
Fit one data point in Ne
everything else is prediction

Q^2 = 3GeV^2 , ν = 16-20 GeV



The ν and Q^2 dependence





Many approximations made!

$$\tilde{D}(z, Q^2, \nu)\Big|_{\zeta}^{\zeta_f} \longrightarrow \tilde{D}(z, Q^2, \nu)\Big|_{\zeta_i}^{\zeta_f}$$

The medium is a bit more complicated in HIC

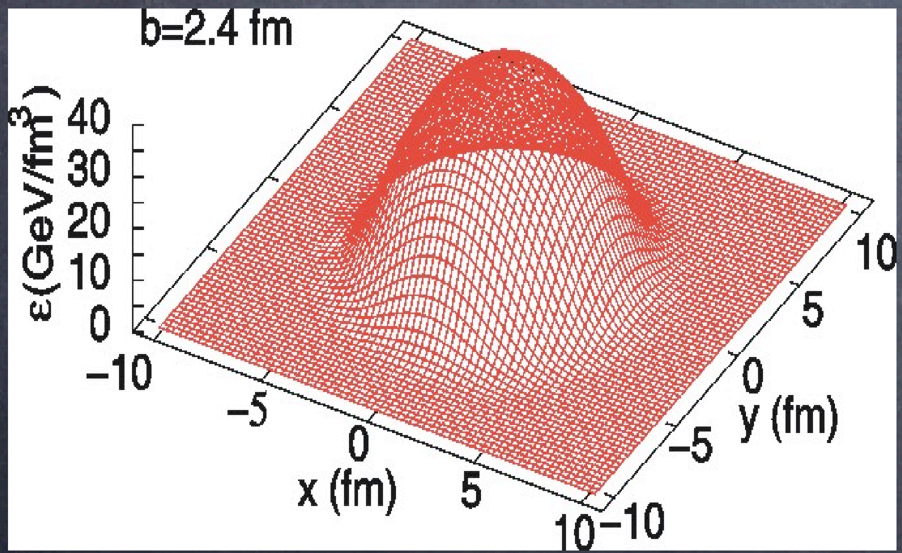
Evolves hydro-dynamically as the jet moves through it

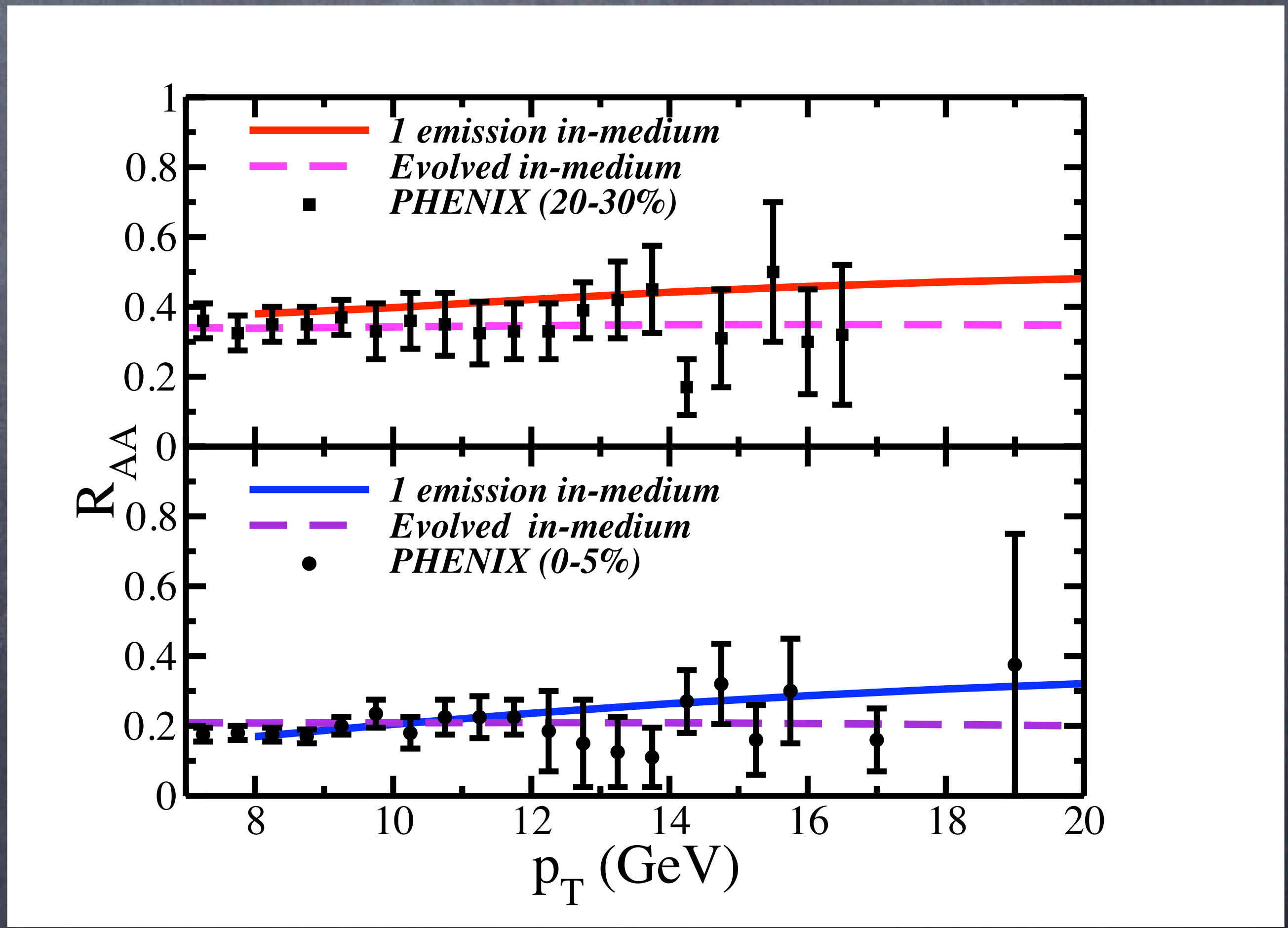
Fit the q̂ for the initial T in the hydro in central coll.

$$\hat{q}(x,t) = \hat{q}_0 \frac{T^3(x,t)}{T_0^3}$$

$$\times \quad [R(x,t) + c_{HG}(1 - R(x,t))]$$

$$\hat{q} = 2GeV^2/fm \text{ at } T = 400MeV$$





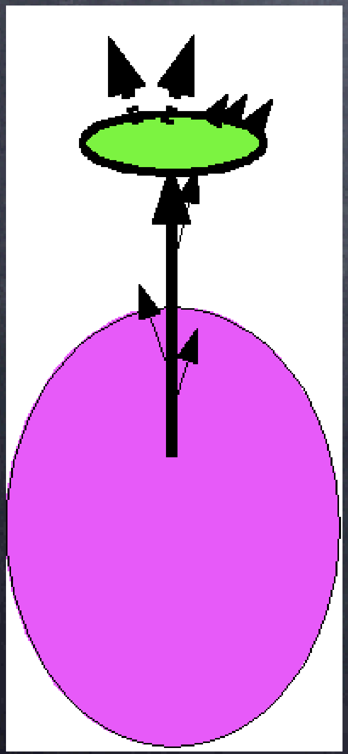
If q̂ scales with ε can make a parameter free estimation of jet quenching in the hadronic phase

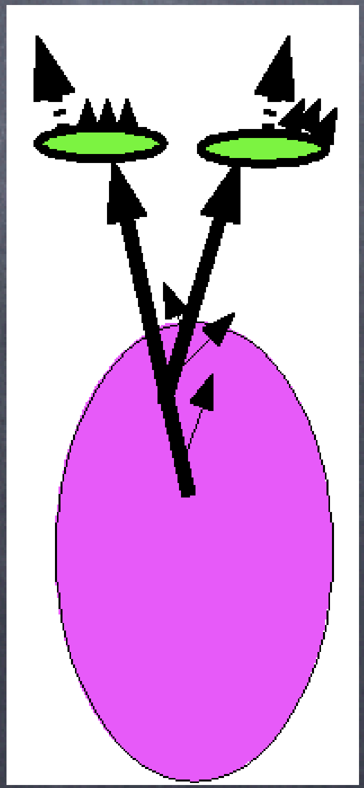
Dihadrons, yet another test of the formalism

Works in DIS with no additional parameters

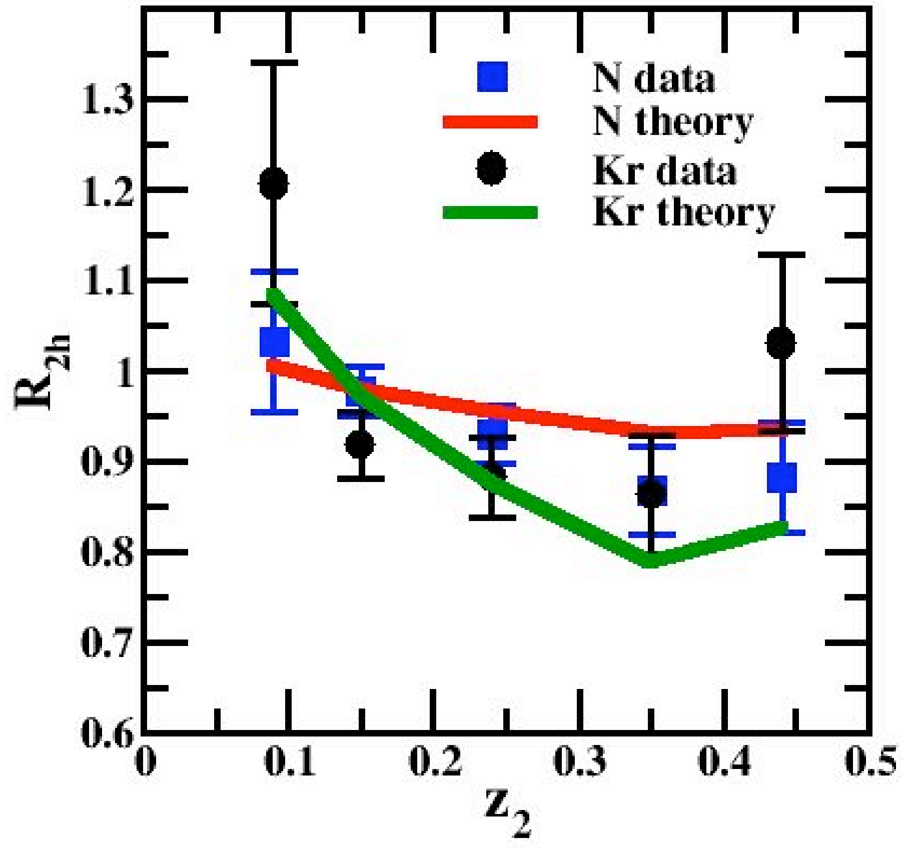
Works in HIC with no additional parameters

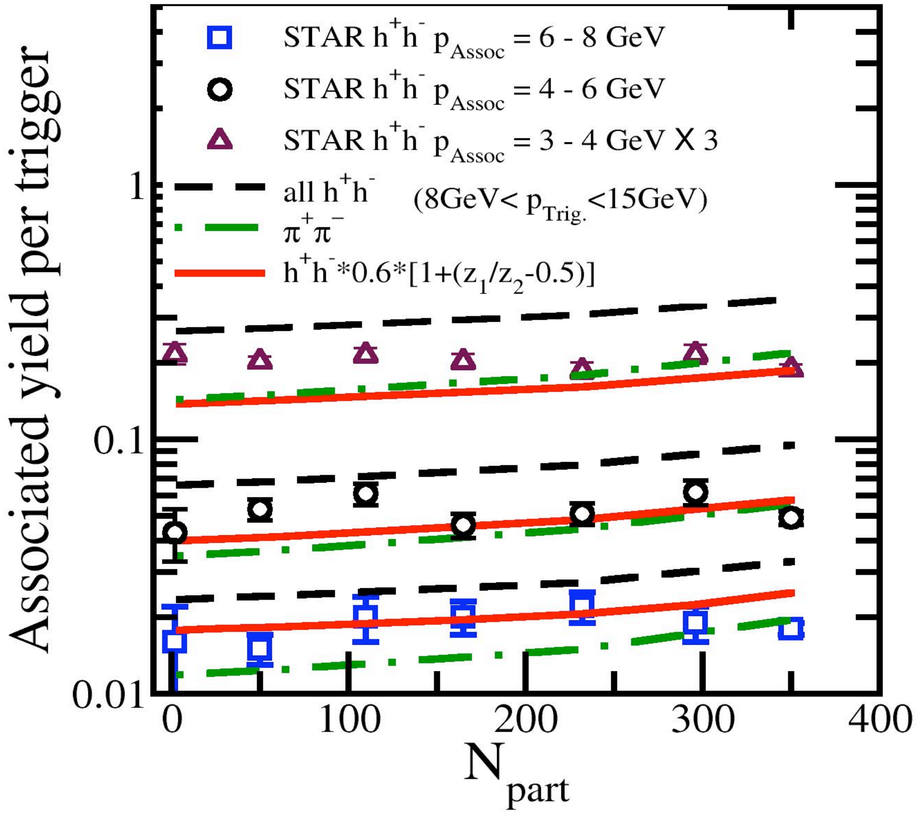
Requires the same non-pert. input a dihadron fragmentation func.





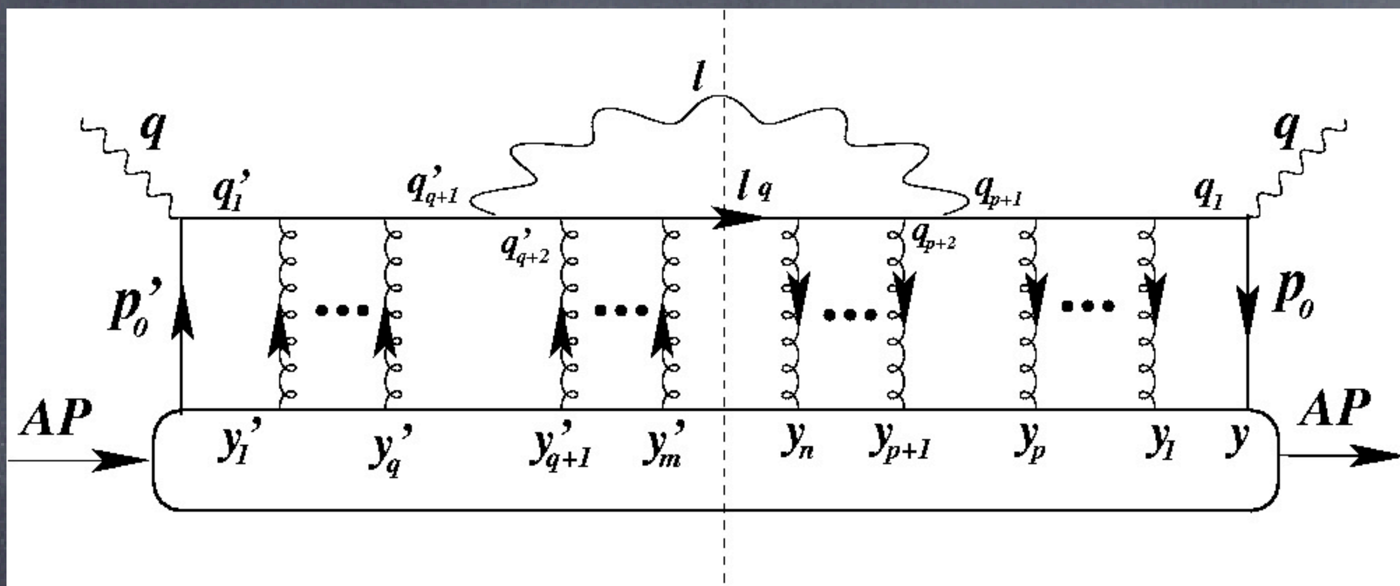
with Wang and Wang

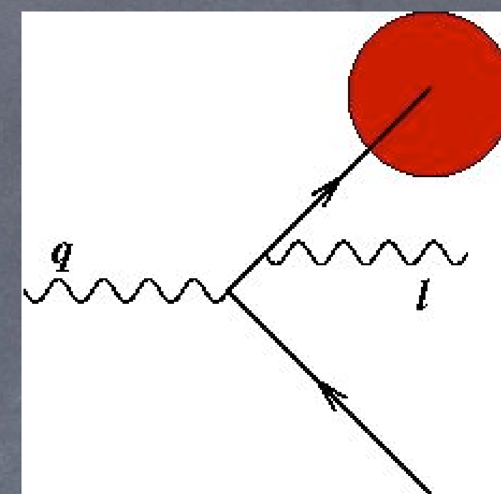


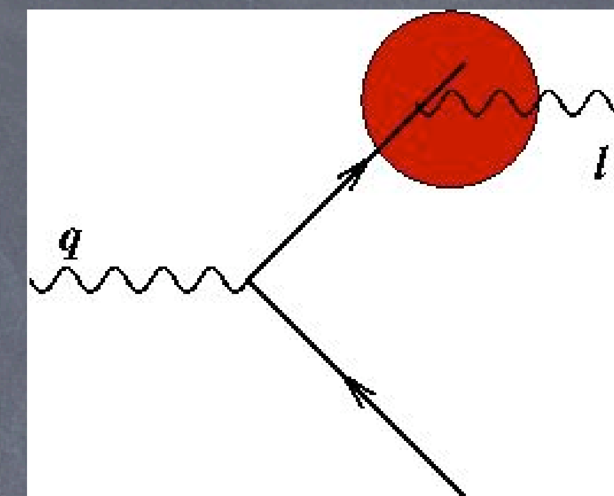


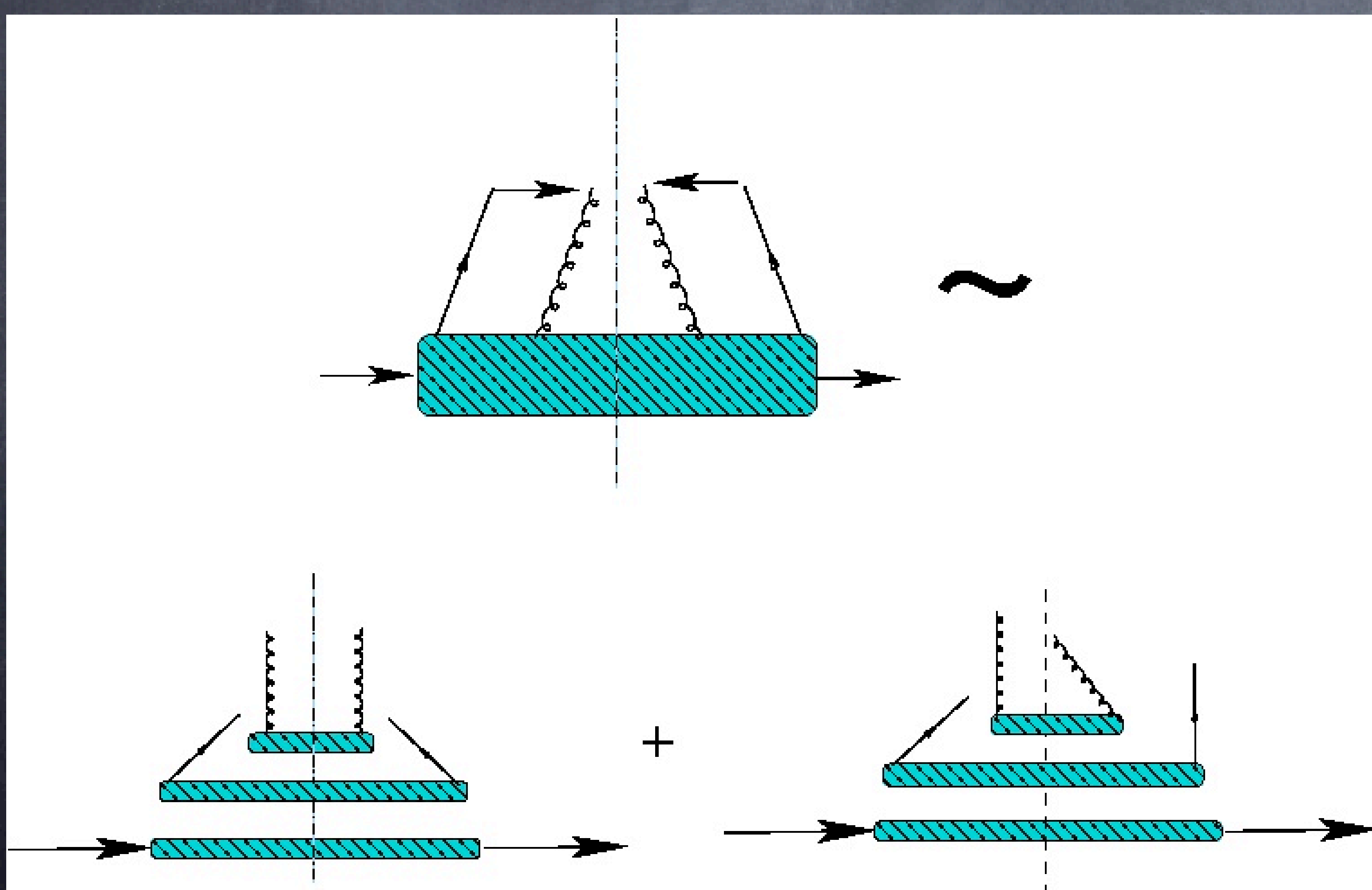
Things you cannot imagine in a HIC

consider photon Brem.









Without a knowledge of the GPD's, E-loss calculations incomplete!

Conclusions: what is missing ?

1) The scale evolution of the transport coefficients

2) Incorporation of elastic loss and diffusion

3) A complete NLO calculation to estimate the error

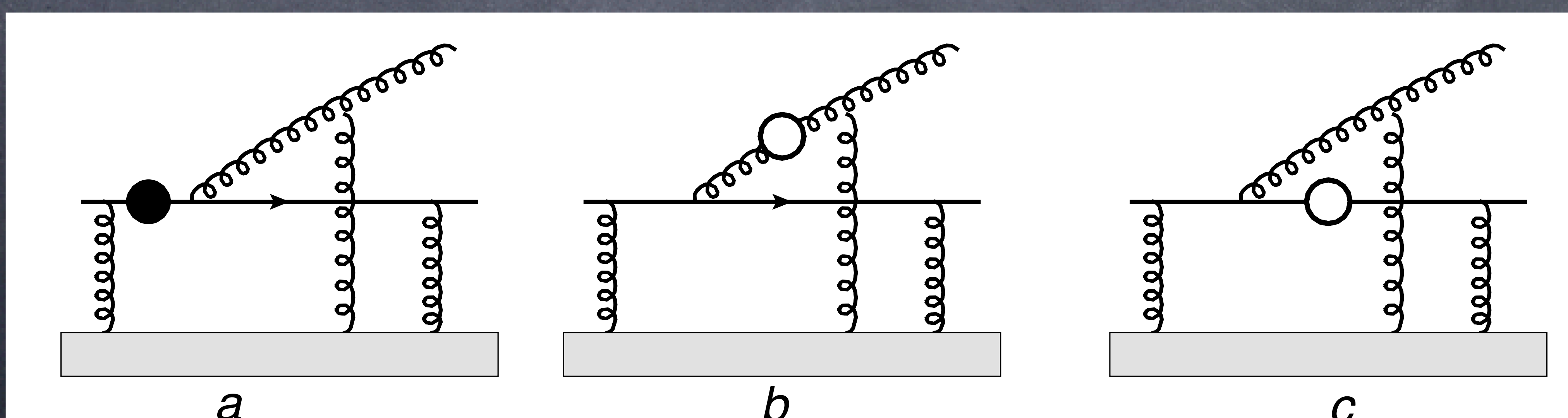
4) Extension to Monte-Carlo simulations (underway!)

6) A first principles calculation of transport coeffs. !?

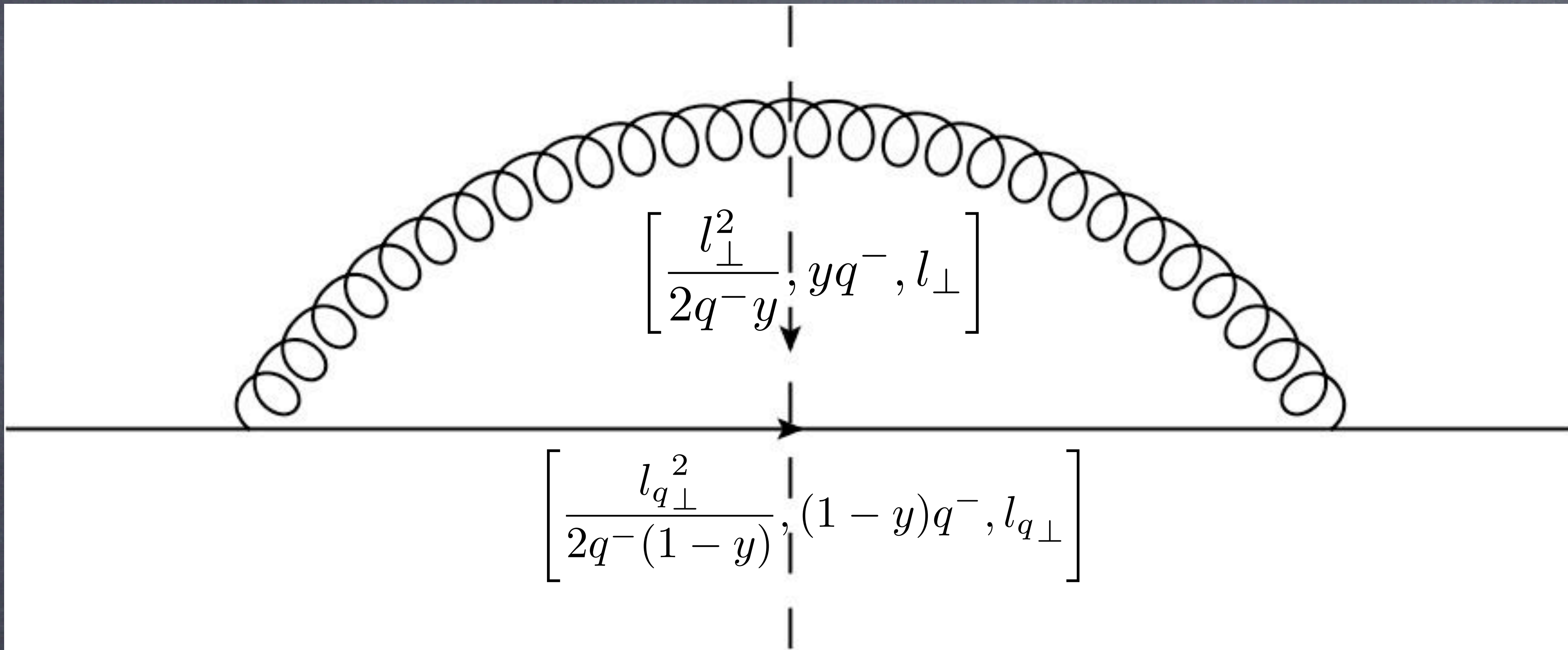
7) Going beyond the lowest order and diagonal coeffs.

The Basic steps:

1) write down the general structure in position space.
2) Fourier transpose all propagators to momentum space
3) assume all k^- are << q^-, integrate. out the k^- .
4) Do as many k^+ integrals, this time orders the locations
5) There will always be one propagator not on shell

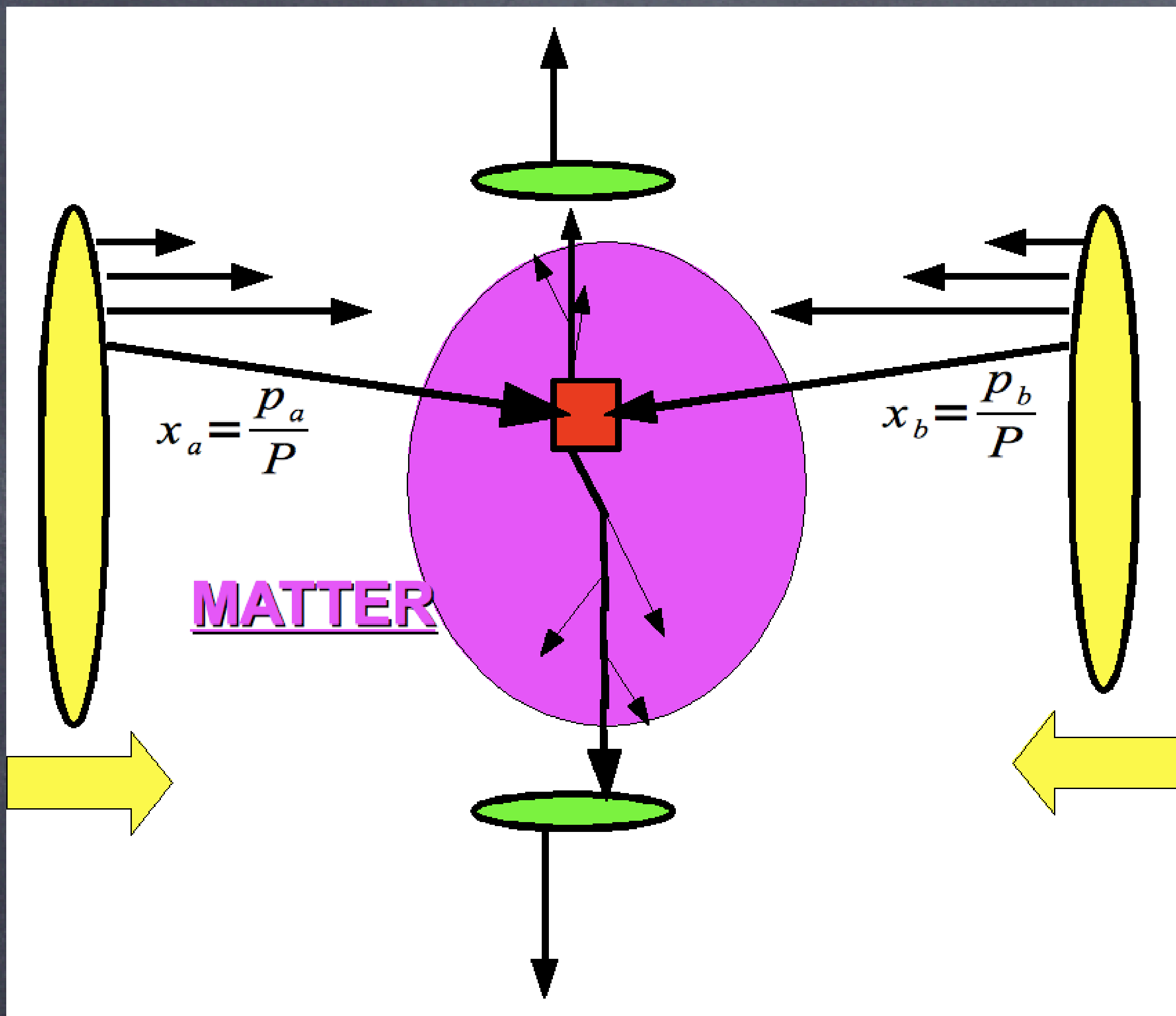


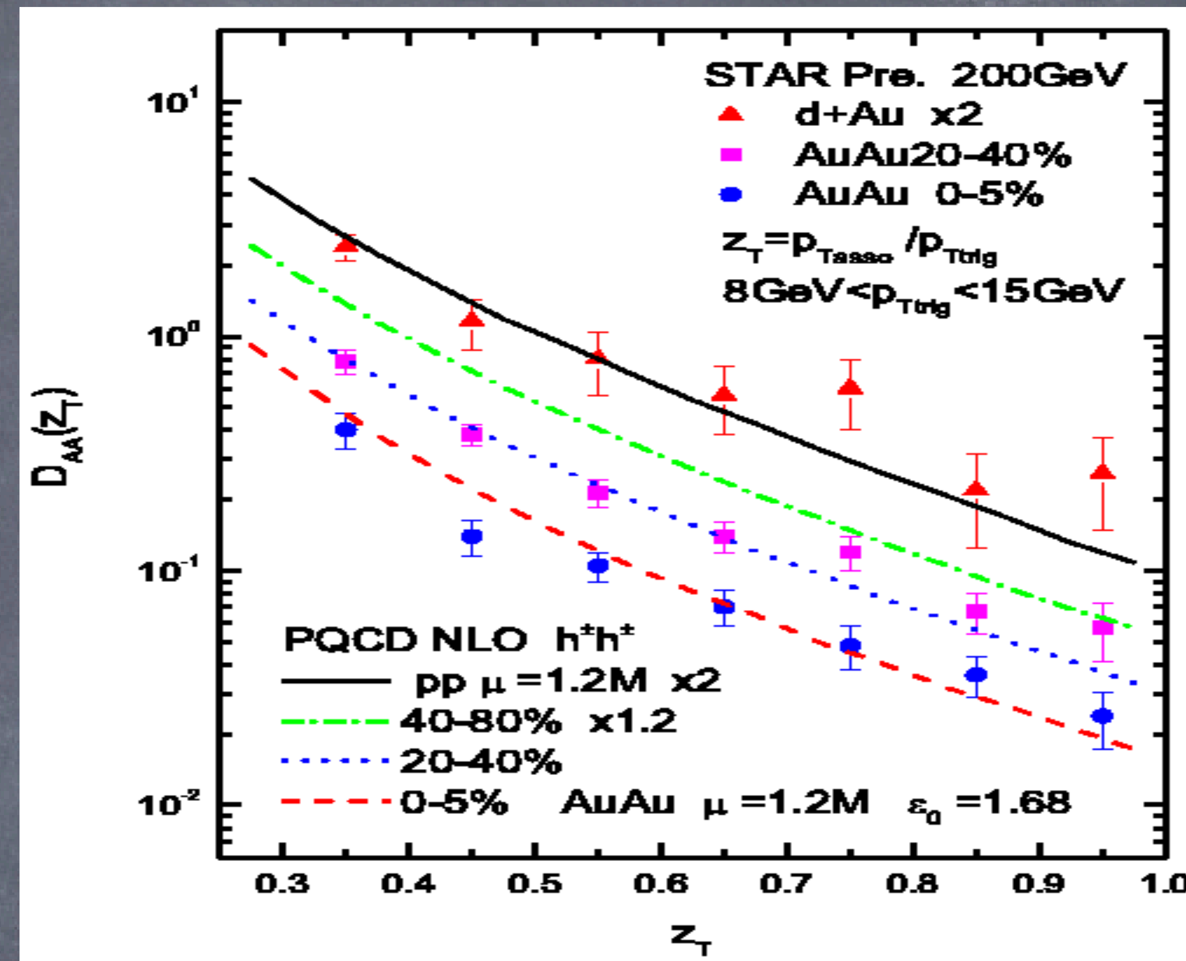
Case of no scattering



$$\sim \frac{\alpha_s C_F}{2\pi} \int dy d^2 l_\perp d^2 l_{q_\perp} P(y) \frac{l_\perp \cdot l_\perp}{l_\perp^2 l_\perp^2} \delta^2(l_\perp + l_{q_\perp})$$

Away side suppression a rigorous consistency check





Zhang, Owens, Wang, Wang

In back-to-back correlations or γ-hadron correlations apply single suppression formalism in a more constrained environment

It would be a serious problem if this did not work!

We can estimate the coeffs. assuming medium is weakly coupled

At leading order in HTL

$$\hat{q} = C_R \alpha_s m_D^2 T \log\left[\frac{4ET}{m_D^2}\right]$$

$$m_D^2 = 4\pi\alpha_s(1 + N_f/6)T^2$$

at T = 400 MeV, E = 20 GeV, α_s = 0.3, q = 2 GeV^2/fm

May be all this scale evolution is for the birds!

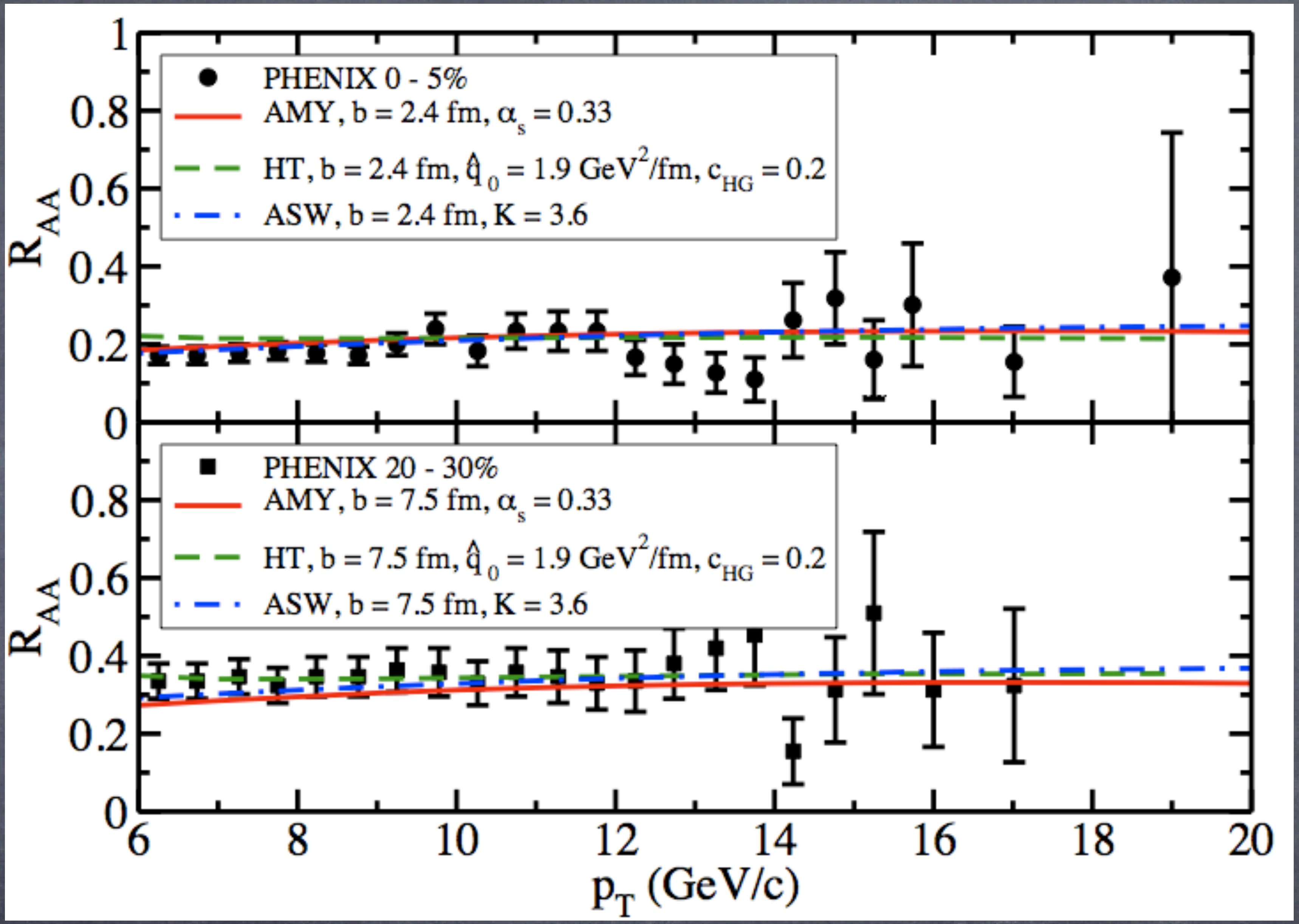
May we can just calculate all transport coeffs.
in LO-HTL and use them

This definitely makes our calculations more predictive

And we all know about this paper!

HT extracted $\hat{q}$ = 4.3 GeV^2/fm ,
AMY $\hat{q}$ from LO-HTL formula with fit α_S = 4.1 GeV^2/fm

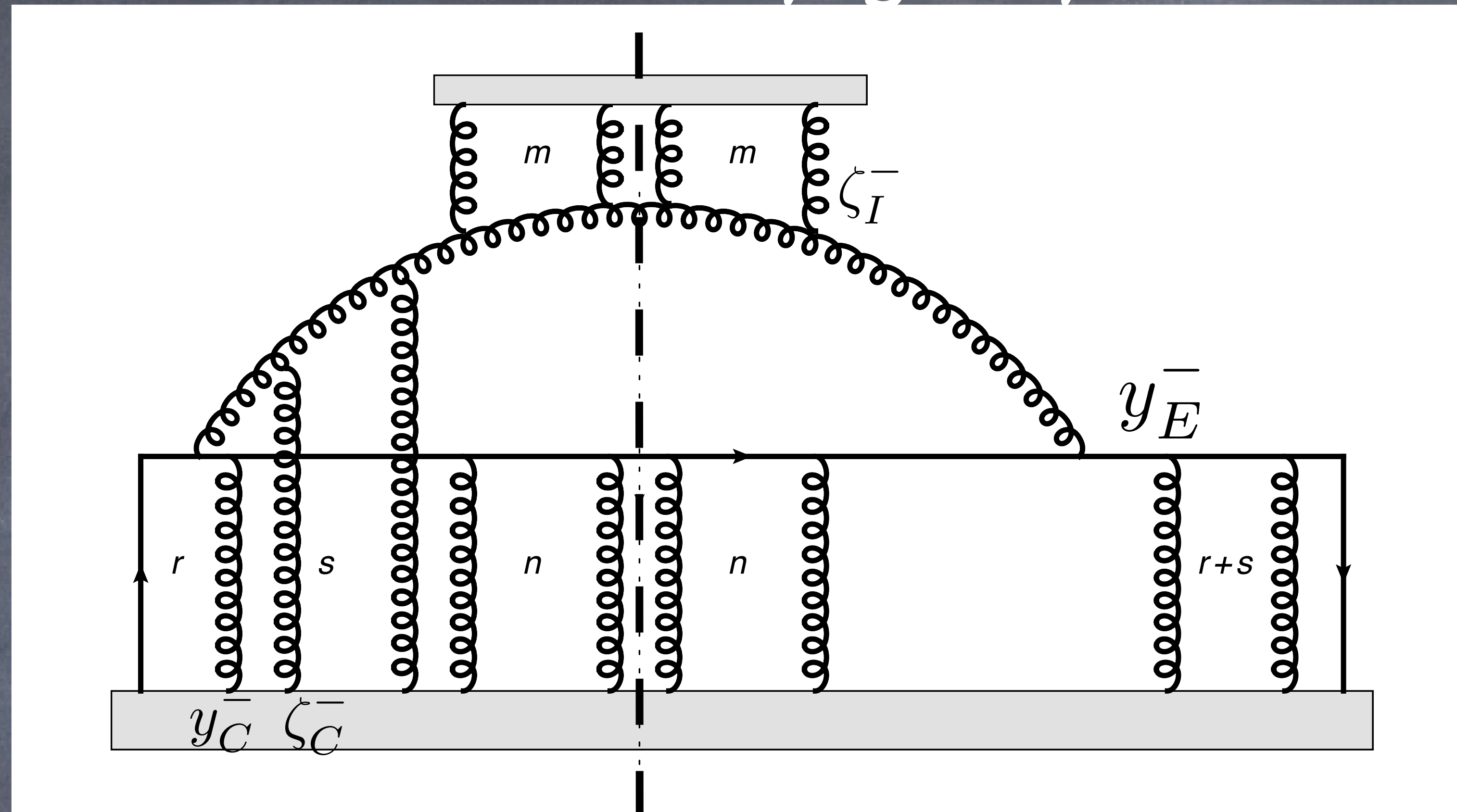
And we all know about this paper!



HT extracted $\hat{q}$ = 4.3 GeV^2/fm ,

AMY $\hat{q}$ from LO-HTL formula with fit α_s = 4.1 GeV^2/fm

The nitty gritty!



$$\int dl_\perp dl_{q\perp} dy \text{ C.F. } \delta^2 \left(q_\perp + l_\perp - \sum_{i=1}^{s} k_\perp^i - \sum_{j=1}^{r} p_\perp^j - \sum_{l=1}^{m} k_\perp^l - \sum_{k=1}^{n} p_\perp^k \right)$$

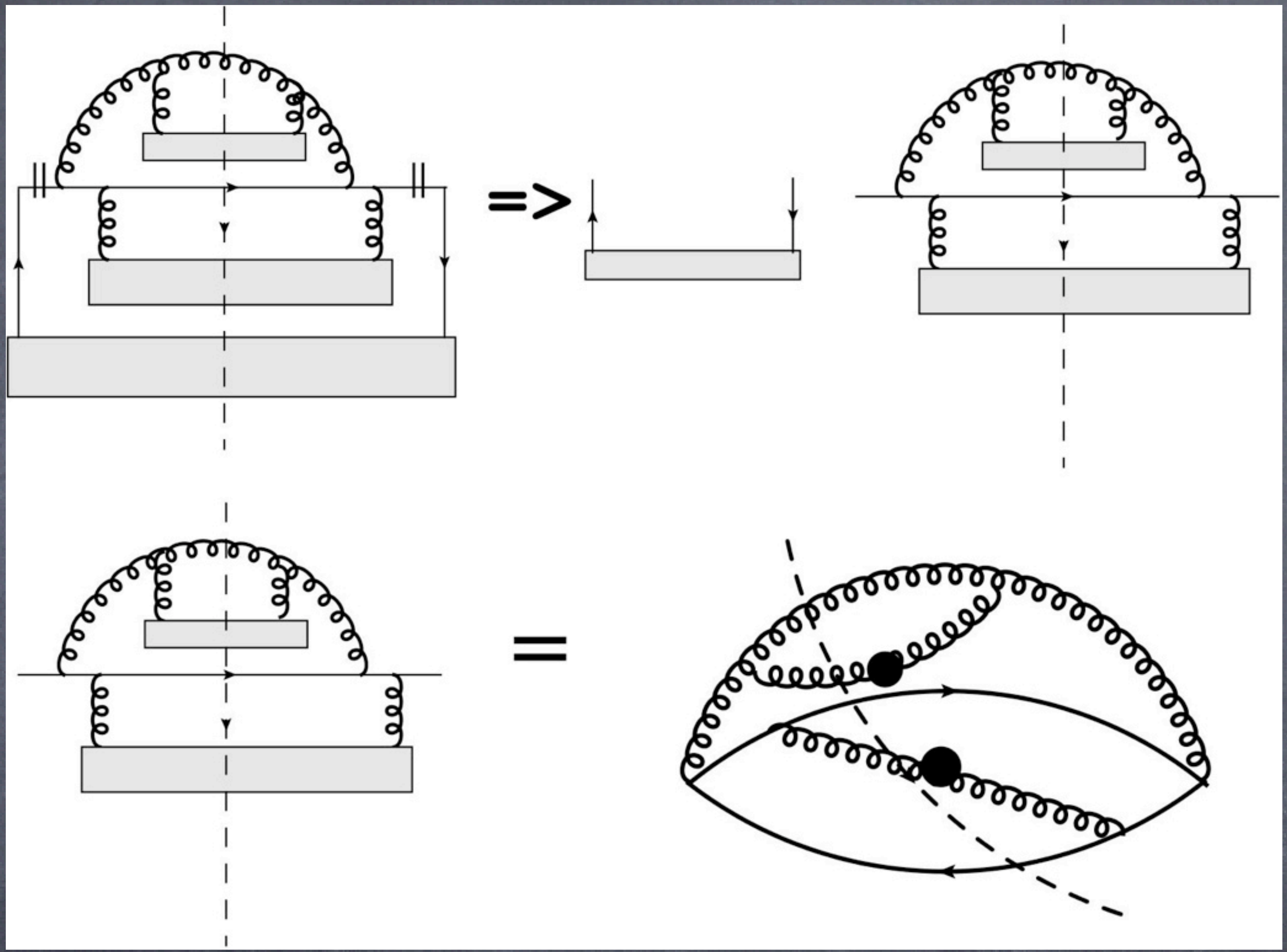
$$\frac{l_\perp - \sum_{i=1}^{s} k_\perp^i - \sum_{l=1}^{m} k_\perp^l}{\left(l_\perp - \sum_{i=1}^{s} k_\perp^i - \sum_{l=1}^{m} k_\perp^l\right)^2} \cdot \frac{l_\perp - y\sum_{i=1}^{r+s} k_\perp^i - \sum_{l=1}^{m} k_\perp^l}{\left(l_\perp - y\sum_{i=1}^{s} k_\perp^i - \sum_{l=1}^{m} k_\perp^l\right)^2}$$

$$\prod_{i=1}^{N} \int dy_i^- \frac{\int d^3\delta y_i \rho \langle p| A^+(y_i^- + \delta y_i^-, 0) A^+(y_i^-, -\delta y_\perp^i)|p\rangle}{2p^+(N_c^2 - 1)} e^{ik_\perp^i \delta y_\perp^i}$$

$$\left[\theta(\zeta_I^- - y_E^-)\left\{e^{-ip^+x_L y_E^-} - e^{-ip^+x_L\zeta_I^-}\right\} - \theta(\zeta_I^- - y_I^-)e^{-ip^+x_L y_I^-} - \theta(y_I^- - \zeta_I^-)e^{-ip^+x_L\zeta_I^-}\right]$$

$$\left[\theta(\zeta_C^- - y_0^-)\left\{e^{ip^+x_L y_0^-} - e^{ip^+x_L\zeta_C^-}\right\} - \theta(\zeta_C^- - y_C^-)e^{ip^+x_L y_C^-} - \theta(y_C^- - \zeta_C^-)e^{ip^+x_L\zeta_C^-}\right]$$

Can compare diagram by diagram with AMY

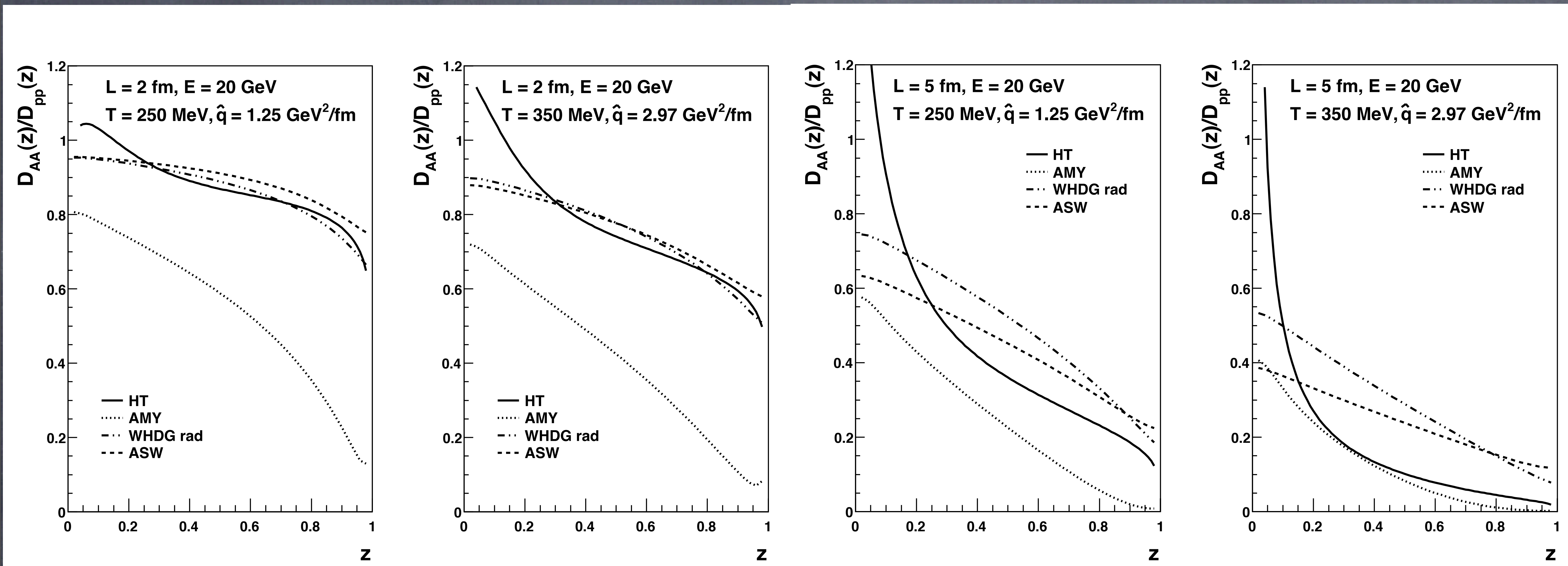


There is an overlap with AMY diagrams

Not all HT contributions are in AMY

@ leading power supp^{rn} in HT, AMY has extra contributions

Problem 1: Its misleading



HT is different from AMY in most cases

AMY has no quenching in hadronic phase

Similarity in $\hat{q}$ between HT and AMY is just coincidence

Problem 2: Heavy quarks

radiative loss is small

Need elastic loss and diff.

All calculated in LO-HTL

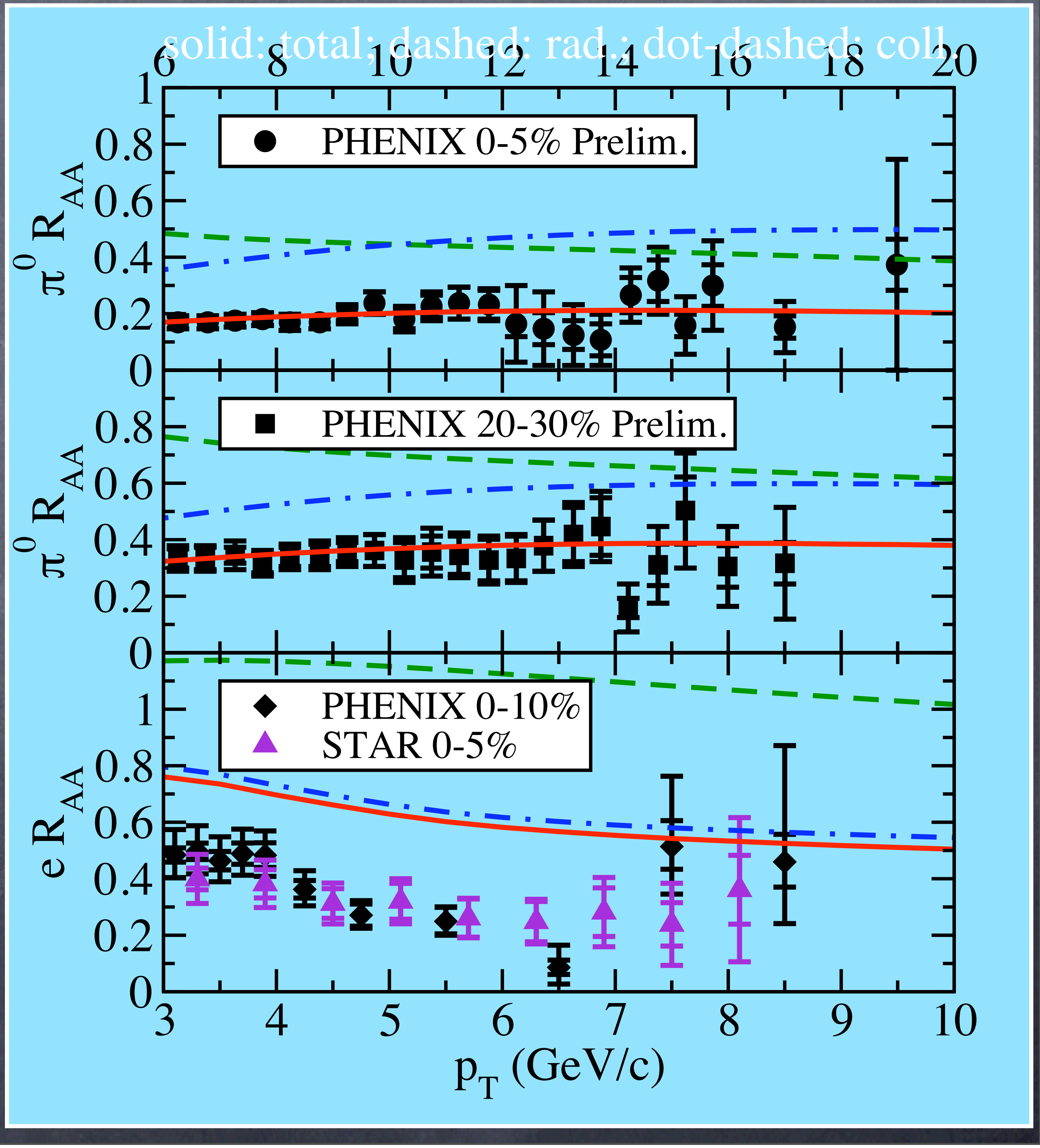
light quarks (Qin,AMY), heavy quarks (Braaten, Thoma)

In agreement with Wicks et. al.

Still really bad for heavys

χ^2/d.o.f. = 222/20 = 11

With G.-Y. Qin



What if medium is not weakly coupled?

$$\hat{q} \propto T^3, \quad \hat{e} \propto T^2 \qquad \hat{e}_2 = \frac{d(\Delta E)^2}{dt}$$

With G.-Y. Qin

This is too much freedom
Lets tie our hands again !

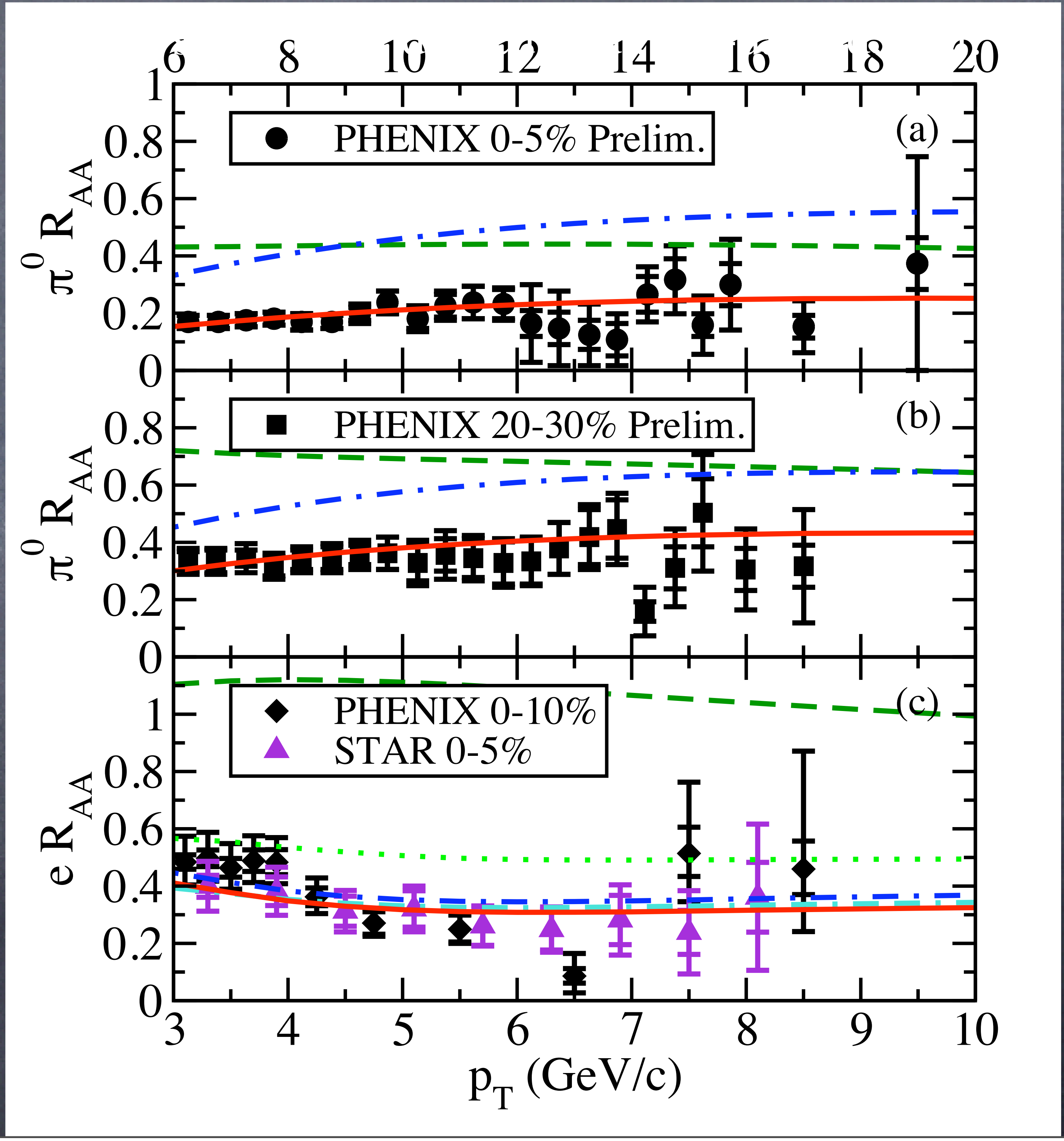
$$\frac{d(\Delta p_\perp)^2}{dt} \simeq 2\frac{d(\Delta p_z)^2}{dt} \simeq \frac{4T}{|v|}\frac{dp_z}{dt}$$

one q for all:
χ^2/dof = 87/20 = 4

2 q's : χ^2/dof = 20/19 = 1

$$\hat{q}_q = 0.7\hat{q}_H$$

$$\hat{q}_q = 0.9\hat{q}_c = 0.6\hat{q}_b$$



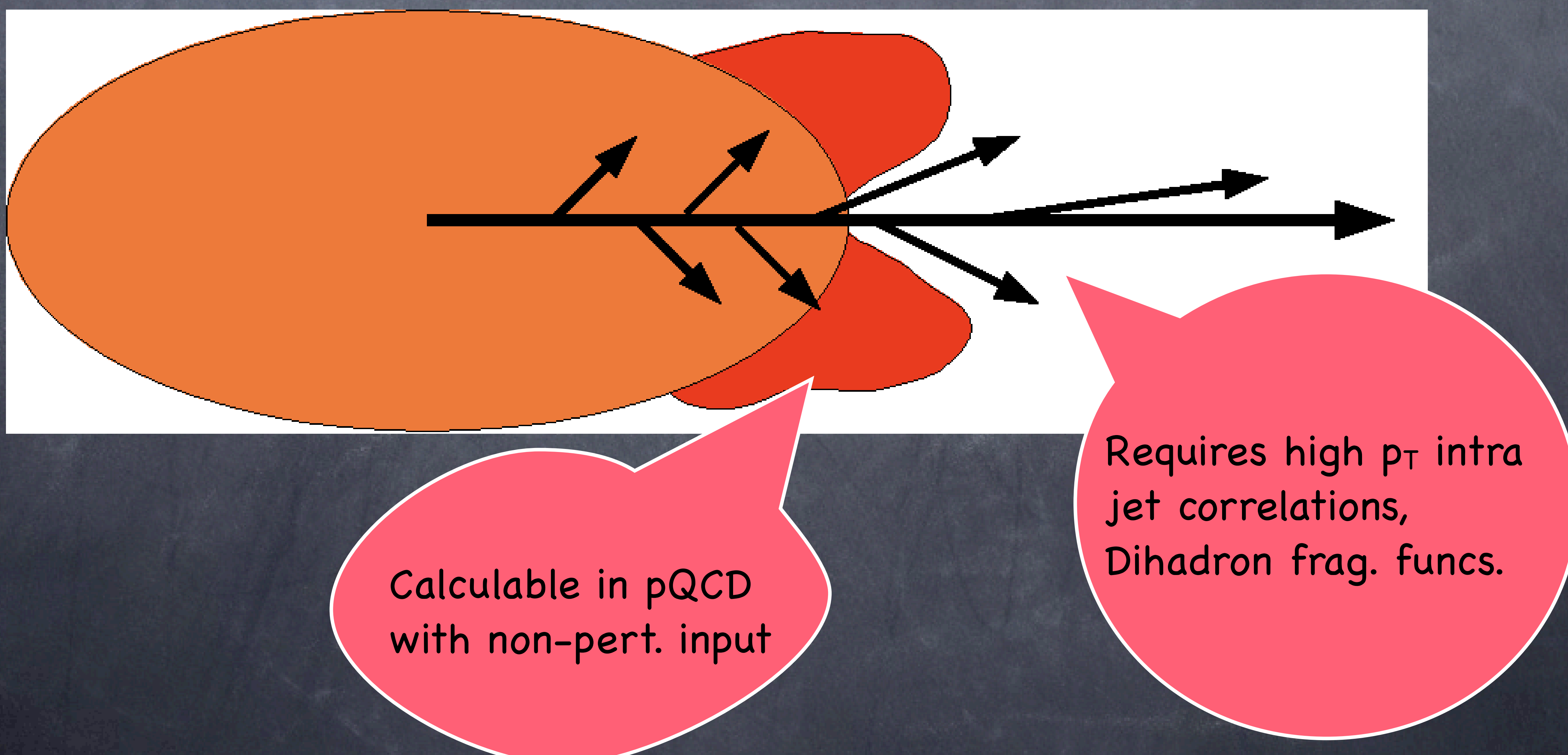
Why larger q for heavier quarks

Near side/intra jet/away side multi-part corr. Require new and extra input

Two related questions:

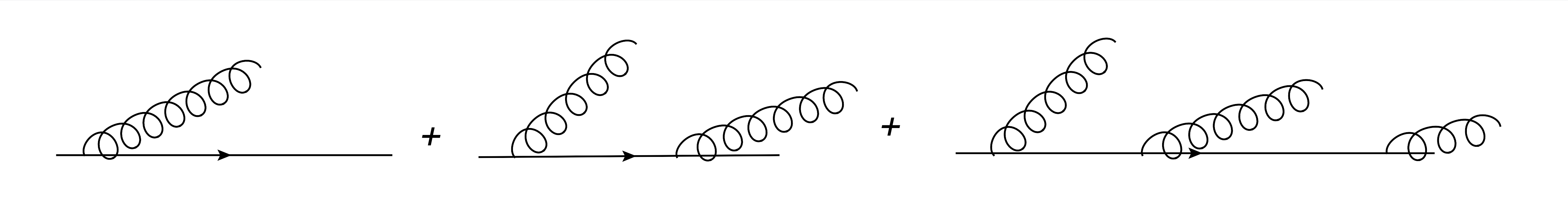
How does the medium change the intra-jet structure

How is the medium changed by energy deposited by jet



Factorization -> evolution

In principle can factorize over a range of scales
Change from one scale to the next involves large logs
Can calculate this re-summation in pQCD, DGLAP evolution



$$\frac{\partial D(z,Q^2)}{\partial \log(Q^2)} = \int_z^1 \frac{dy}{y} P(y) D\left(\frac{z}{y}, Q^2\right)$$

Applicability of pQCD means calculating the c.s. and the evolution of soft quantities.

Leading log from leading pole and interpretation

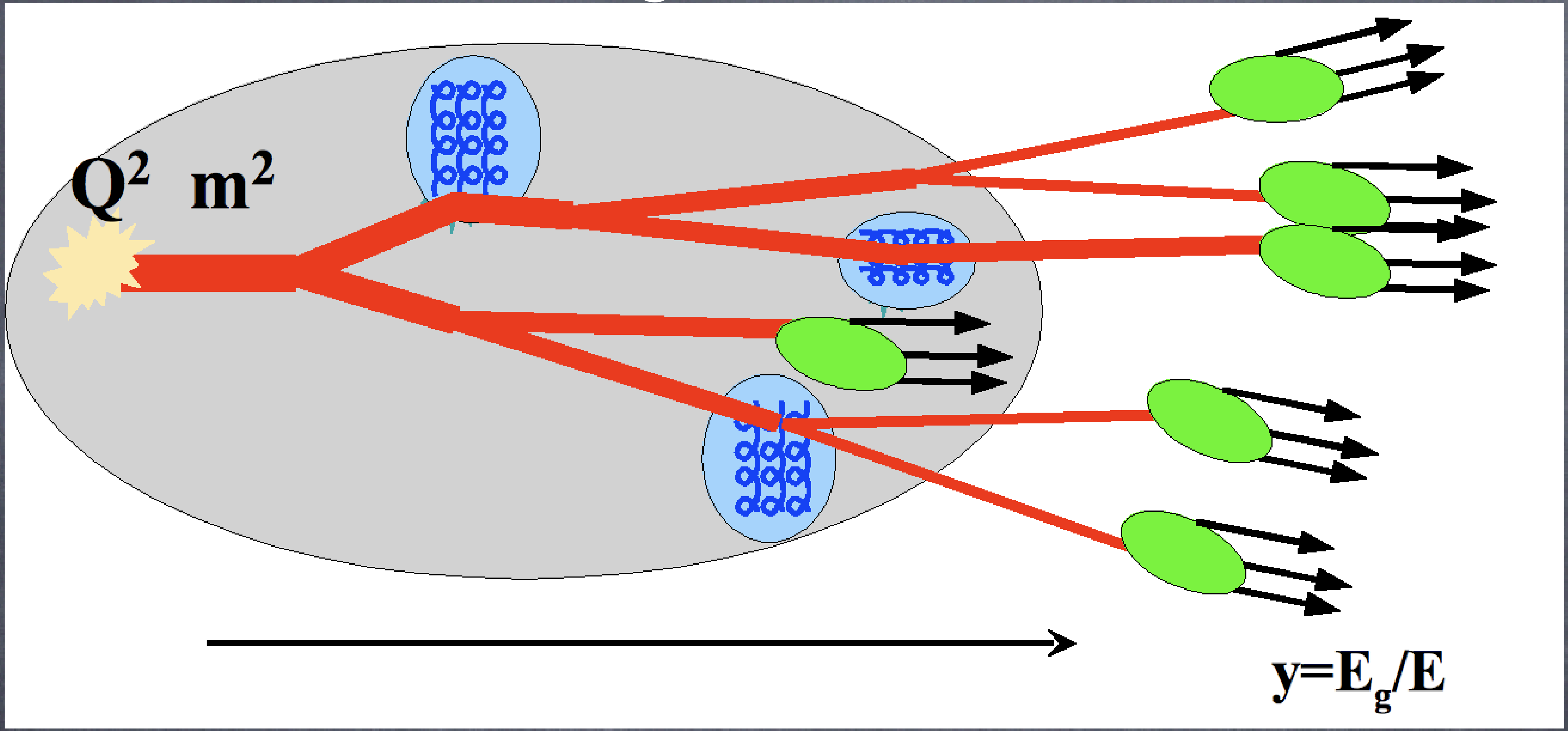
In a standard DGLAP kernel calculation
the final states are considered on shell

This only means that they travel far compared to the current inverse mass scale

Thus have smaller virtuality compared to the parent

The final states still have a $Q^2 >> \Lambda_{QCD}{}^2$

Extending this to the case of DIS on a large nucleus.

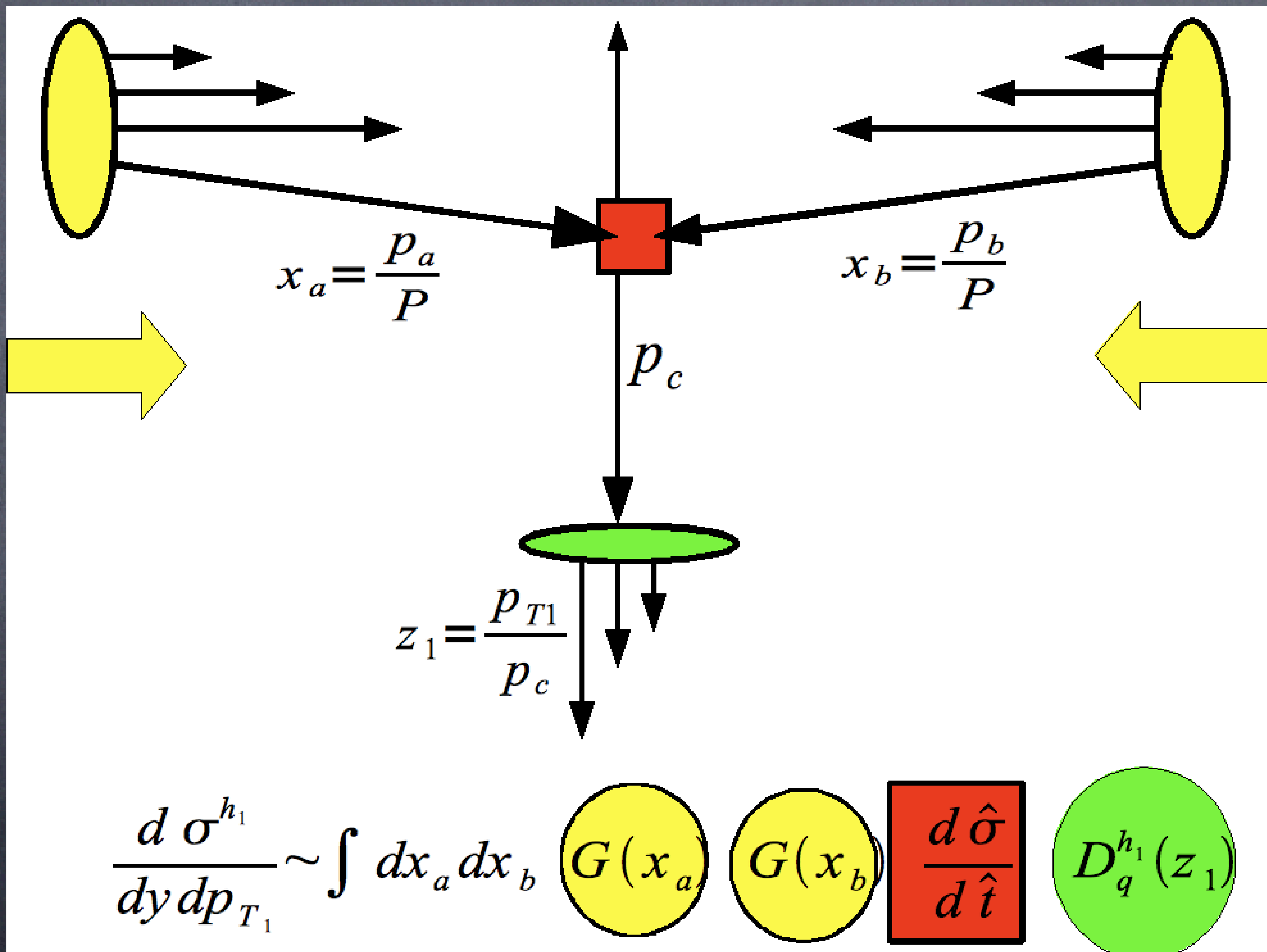


For a few particles use frag. func. formalism

$$D + (\text{calculated } \#) * D = \tilde{D}$$

The large nucleus is a space filler for a medium
In a factorized formalism, replace this with any medium

Given factorization and universality,
Can connect e^+e^-, DIS, pp and HI collisions



Can you sum this series ?

$$1 - x^2 + \frac{1}{2}x^3 + \frac{1}{3}x^4 - \frac{11}{24}x^5 + O(x^6)$$

$$x \gg 1$$

Can you sum this series ?

$$1 - x^2 + \frac{1}{2}x^3 + \frac{1}{3}x^4 - \frac{11}{24}x^5 + O(x^6)$$

$$x \gg 1$$

$$e^{-x}\left(1 + x e^{-x} + \frac{1}{2}x^2\left(e^{-x}\right)^2 + \frac{1}{6}x^3\left(e^{-x}\right)^3 + \frac{1}{24}x^4\left(e^{-x}\right)^4 + \frac{1}{120}x^5\left(e^{-x}\right)^5\right)$$